

**MATHEMATICAL MODELING OF SOIL EROSION PROCESSES
USING A PHYSICALLY-BASED AND EMPIRICAL MODELS:
CASE STUDY OF SLOVAKIA AND CENTRAL POLAND**

Zuzana Némětová*, Silvia Kohnová

The article is focused on mathematical modelling of the soil erosion processes on the selected areas within the Slovak Republic and Poland. The study includes the validation of the models used based on the actual measurements. The intensity of the soil erosion processes was calculated using the physically-based EROSION-3D model and the empirical USLE-SDR model. The simulations were done based on continuous rainfall events and a long-term simulation. The results modelled were confronted with the actual measurements in both areas investigated. Since a model's validation and calibration as well as a relevant interpretation of the results obtained are the hardest and most challenging parts of any research, it is necessary to constantly enhance the techniques and methods of the calibration and validation of models, thereby deepening the knowledge of individual models. The results show the process has to be performed before the application of the models used together with the advantages and disadvantages of the physically-based and empirical models used, including a comparison and validation of the models applied.

KEY WORDS: mathematical model, validation, calibration, erosion, extreme rainfall event, physically-based model

Introduction

A basic and fundamental definition of soil erosion is a process involving the detachment, transport, and accumulation of soil materials from any part of the Earth's surface. Since soil erosion occupies the most serious position among the individual degradation processes, it plays an irreplaceable role in scientific research (Rawat et al., 2011; Markantonis et al., 2012). Soil is one of the fundamental elements that provide resources for human food supplies; therefore, the significance of soil erosion research lies in an understanding of soil erosion processes over large areas of the Earth (Wainwright et al., 2003).

Since soil erosion research by the Soil Erosion Service of the US Department of Agriculture began around in 1930 (Bennet, 1933), a large number of soil erosion and sediment transport models have been developed, but choosing a suitable model that fits the needs of individual users is still a complicated and problematic task. The models are a useful tool to simulate nature, but they suffer from a range of problems, such as overestimation due to the uncertain results of the models or difficulties in obtaining input data (Hajjizadeh and Hector, 2018). It is important to note that the modelling of nature is always constrained by many differences in the sense of spatial and temporal variability, spatial heterogeneity,

the transport media, and the high instability of the input data (Jakeman et al., 1999). Nature is and always will be beyond us.

The most influential factors that impact water erosion are the climate, topography, soil structure, vegetation, anthropogenic activities, and management systems used (Kuznetsov et al., 1998). All of these factors can influence various elements of sediment and erosion processes along with deposition as well. It is assumed that the major triggers of splash and sheet erosion are rainfall intensity and the runoff rate (Wei et al., 2009). These two factors can be very powerful together with the human activities in catchments. Inappropriate anthropogenic activities that cause changes in the magnitude and nature of the material inputs to estuaries can also involve erosion with consequences for populations and ecosystems (Iglesias et al., 2019).

The degree of sedimentation in a catchment is heterogeneous in space and over time, depending on the land use, soil type, topography, any slopes, vegetation cover, and the climate (Marttila and Klöve, 2010).

The estimation of erosion and sediment processes can be modelled by different kinds of models, i.e., empirical models, physically-based models or conceptual models. The selection of the methods used depends on the size of the study area, the objectives of the research, the available input data or the possible validation and

calibration of the models (García-Ruiz et al., 2015). The selection of a model is a very important step, but the possibility of validating and calibrating it to the specific catchment conditions is a necessary part of the overall success of the modelling. There are several methods established for the validation of the models used, e.g., the bathymetric measurement of sediment deposited in a reservoir has been presented as a suitable method for assessing the volume of eroded material in a research area.

The modelling of the amount of sediments is a useful approach for quantifying the historical impact of agriculture on soil erosion and sediment yields, as well as a good method for calibrating and testing the erosion models in contrast to the actual measurements (Boyle et al., 2011).

The aim of this study is the application of physically-based and empirically-based erosion models in two different countries, i.e., the Slovak Republic and Poland, together with a comparison of the models with the actual measured data in both research areas investigated.

Material and methods

The subchapter describes the methods used and the areas under research. Because two different countries were investigated, the chapter is divided into two main parts. The first contains a description of the Svacenický Creek research area in Slovakia, and the second describes the investigation site of the Zagożdżonka catchment in Poland.

Svacenický Jarok research area (Slovak Republic)

The area investigated is located in the western part of Slovakia (Fig. 1) in the area of the Myjava Uplands.

The whole catchment area is prone to intensive erosion processes and quick runoff responses, which are the results of a massive 600-year transformation from a natural to an agricultural landscape (Fig. 1a). The current composition of the land use is as follows: arable land covers 66% of the area, while forests occupy 9%, grasslands 9%, water bodies 7%, gardens 6%, buildings 2%, and shrubs 1% of the catchment. The Svacenický stream passes through the area as a right-hand tributary of the Myjava River. The creek flows into a small water reservoir (polder) in the lower part. The town of Myjava has been impacted by frequent floods in the past (Stankoviansky, 2003; Dotterweich et al., 2013), and the construction of the polder was necessary in order to ensure flood protection in terms of the reduction of flood flows.

Parameters input and field terrain survey of the Svacenický Jarok

For the modelling of the soil water erosion, physically-based EROSION-3D model was used. In the case of the model's inputs, the rainfall data, relief parameters, and soil input characteristics were needed. The rainfall data were obtained at the Myjava meteorological station during the period evaluated (Figs. 4 and 5). Information about the soil was obtained during the field measurements. The relief parameters are represented by a digital elevation model (Fig. 1b). A long-term simulation was performed with the EROSION-3D model based on continuous rainfall events. The results from the model were compared with the bathymetric measurements in order to perform the validation of the physically-based EROSION-3D model. More information about the bathymetric measurements can be found in Némětová et al. (2020), Honek et al. (2020).

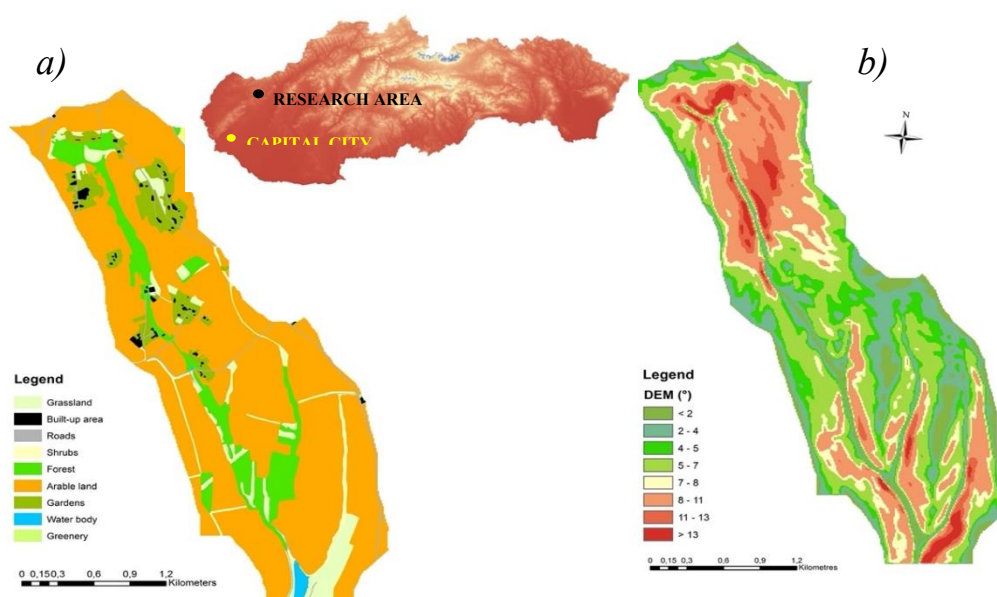


Fig. 1. Localization of the Svacenický Jarok, Slovak Republic research area; a) Land use, b) Digital elevation model.

The measurement of a terrain represents a necessary step in order to determine the adequate soil input parameters and therefore ensure the relevant outputs. A field survey and soil sampling in the Svacenický Jarok were carried out in the summer of 2018 in cooperation with Masaryk University in Brno. The measurement of the terrain was performed not only with the aim of taking and evaluating the soil samples, but also to enable interchanges with local experts in order to identify the crops cultivated in the period from 2012 to 2018. Laboratory analyses were conducted for the selected soil input parameters in the laboratory of the Slovak University of Technology in Bratislava and Masaryk University in Brno. An example of the results analysed is shown in Table 1.

Sensitivity analyses of the soil input parameters

Sensitivity analyses represent a useful element of the evaluation of every model. The analyses include the process of changing the input values with the identification of the impact of these changes on the final results. The goal of the sensitivity analyses consists of knowledge of the quantification and identification of a model's inputs and outputs as well as an understanding of the model's relationships. The analyses should be done before the application of every model in order to see how the parameters input influence the individual outputs. Here, the soil input parameters were increased and decreased by about 10% in comparison with the reference (initial) state. The sensitivity analyses were performed in previous studies; for more information, see Honek et al. (2020) or Németová et al. (2020).

Methodology of creating a connection between the plots and a map of the land use

It was necessary to connect the individual plots (LPIS parcels; LPSI – agricultural areas with stable natural or artificial boundaries) with the current land use according to the direct years (2012–2018) and therefore ensure the association between the plots and the land use maps regarding the individual years (Table 2). A new land-use map was created for each year, where the particular areas were divided into parcels according to the LPIS. The arable land does not represent one homogenous unit but is composed of different elements formed by defined LPIS plots with the cultivated crops for the years 2012–2018. This means that each area, i.e., parcel, has its own ID (in GIS), which corresponds to the specific crop (Fig. 2, Table 2). The dominant crops for the selected years together with the amount of precipitation are shown in the Table 3.

Zagożdżonka (Poland) research area

The second research area (Zagożdżonka catchment) is located in central Poland, approximately 100 km south of the capital city of Warsaw in the Mazovia Lowlands (Fig. 6). The catchment is characterized by a lowland disposition and topography typical of this part of Poland. Agricultural production is dominant, with arable land covering 48% of the area. The forests cover 39% of the catchment, and the pastures occupy the remaining 13%. The localization of the catchment is shown in Fig. 6.

Table 1. Results of the terrain measurements and laboratory analyses (bulk density and organic carbon content)

Designation of the soil sample	AMSL [m]	N [°]	E [°]	Weight before drying [g]	Bulk density reduced [g cm ⁻³]	Organic carbon content [%]
1	310	48°45'17"	17°33'4"	220.9	1.1121	9.5
2	310	48°45'14"	17°33'4"	230.7	1.2599	12.5
3	310	48°45'10"	17°33'2"	258.6	1.4828	11.8
4	310	48°45'10"	17°32'58"	239.5	1.2707	8.8
5	310	48°45'16"	17°32'58"	245.5	1.408	9.4
6	360	48°45'33"	17°32'37"	215.5	1.07	9.2
7	340	48°45'34"	17°32'39"	208.5	1.1605	10.7
8	330	48°45'35"	17°32'40"	194.7	1.0315	15.1
9	420	48°46'52"	17°31'30"	198.9	1.0965	12.1
10	410	48°46'52"	17°31'32"	188.0	1.0109	12.8
11	370	48°46'53"	17°31'34"	238.9	1.4657	10.9
12	410	48°47'3"	17°31'30"	235.4	1.0588	10.1
13	410	48°47'3"	17°31'30"	208.5	0.8895	9.7
14	420	48°47'13"	17°31'25"	228.3	1.334	8.6
15	430	48°47'14"	17°31'29"	183.9	1.0475	7.4
16	450	48°47'17"	17°31'34"	174.7	0.9818	6.9

Staw Górny water reservoir

The Staw Górny water reservoir was constructed in 1976 in order to provide water for a local chemical factory. The total area of the reservoir is 14 hectares; the original volume of the reservoir was 252,000 m³ at a water level of 146.70 m.a.s.l. After 10 years (1986), the water level increased about 42 centimetres. Because of intensive erosion activity in the Zagożdżonka catchment, sedimentation represents a major problem to be dealt with. Several measurements of the water level were performed; i.e., the first measurements of the water reservoir were taken in 1979–1980 based on the Range Line method (Banasik and Mordziński, 1982) and then from 1991–2003 (Banasik et al., 2005). The last survey was performed based on the hydrographic system in 2009 (Banasik et al., 2001; Banasik, et al., 1995).

Input parameters (Zagożdżonka research area)

The simulations were conducted using the physically-based EROSION-3D model and USLE-SDR empirical model. The USLE-SDR empirical model represents a suitable approach for determining the amounts of sediment in the areas under consideration. The results

from both models used were contrasted with the terrain measurements of the sediments in the Staw Górny reservoir. All the input parameters required for the models (the EROSION-3D and USLE-SDR models) were provided by the University of Warsaw under the COST program (CA 16209). The EROSION-3D model requires three input parameters (relief characteristic in the form of digital elevation model, soil input parameters and rainfall data). The equation of empirical model USLE-SDR consists of three components (sediment removal ratio, annual soil loss per unit area and catchment area) and the parameters were estimated for the Zagożdżonka catchment using topographic maps, soil and land maps in the previous studies (Banasik et al., 1995; Banasik et al., 2005). A summary of the rainfall events used in the EROSION-3D model is displayed in Fig. 7.

Results and discussion

Comparison of the results modelled with the actual measurements, Svacenický Jarok, Slovak Republic

The validation of the EROSION-3D model was performed on a continuous series of precipitation

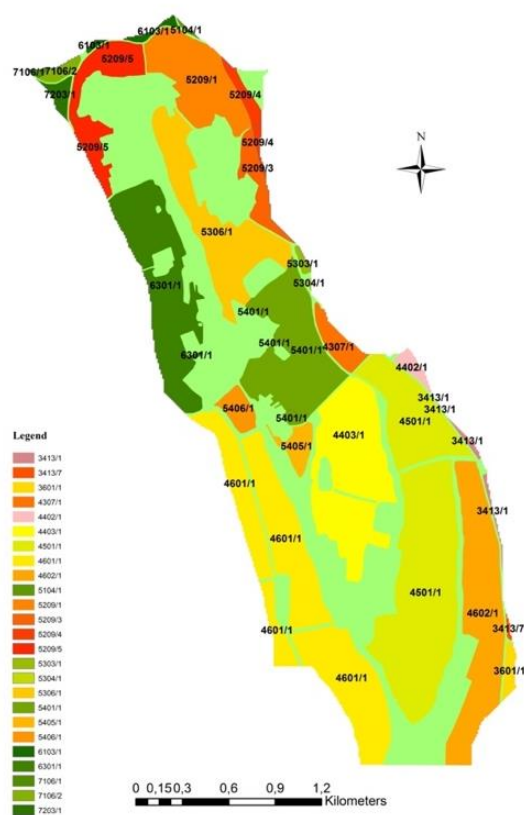


Fig. 2. Land Parcel Identification System (LPIS), Svacenický Jarok, Slovak Republic = agricultural areas with stable natural or artificial boundaries.

Table 2. Identification of the crops according to the individual parcels

Parcel number	2012	2013	2014
3413/1	sorghum	lantern	sorghum
3413/7	wheat	barley	beet
3601/1	wheat	barley	beet
4307/1	sorghum	alfalfa	sorghum
4402/1	sorghum	alfalfa	sorghum
4403/1	beet	wheat	rye
4501/1	wheat	wheat	rye
4602/1	wheat	barley	beet
5104/1	corn	wheat	corn
5209/1	corn	wheat	corn
5209/3	corn	wheat	corn
5209/4	corn	wheat	corn
5209/5	alfalfa	alfalfa	corn
5303/1	sorghum	alfalfa	sorghum
5304/1	sorghum	alfalfa	sorghum
5306/1	sorghum	wheat	wheat
5401/1	beet	wheat	wheat
5405/1	corn	wheat	corn
5406/1	corn	wheat	corn
5601/1	corn	wheat	rye
6103/1	alfalfa	alfalfa	alfalfa
7106/1	alfalfa	alfalfa	alfalfa
7106/2	alfalfa	alfalfa	alfalfa
7203/1	alfalfa	alfalfa	alfalfa

measured at the Myjava meteorological station and on the basis of bathymetric measurements of sediments in the Svacenický Creek polder. In the case of the rainfall events used, one-minute events were selected, which are considered to be erosively effective (Renard et al., 1997). For each precipitation event, a specific set of soil characteristic parameters was defined that corresponded to the date of the occurrence of the precipitation. The final results of the modelling with the EROSION-3D model are displayed in Fig. 8. The results reflect the two periods evaluated, i.e., 2013 (Fig. 8a) and 2014 (Fig. 8b). Fig. 8a represents the year 2013 with the land use composition shown in Fig. 3a. The predominant part of the area represents winter wheat, which is generally considered to be a protective crop. The intensity of the sediments and erosion processes are distributed throughout the whole territory (with more influence on the right part) in comparison with scenario B, where the most intensive impact of the soil erosion processes is related to the arable land represented by corn and rye (Fig. 3b). In the case of the predicted amounts of sediments, the year 2013 (Fig. 8a) was closer to the measured amounts of sediment in comparison with the year 2014 (Fig. 8b). A higher amount of the sediments modelled was predicted in the year 2014, even though the land use structure is composed of rye. On the other

hand, the most intensive processes were detected in the year 2014.

Comparison of the results modelled with the actual measurements, Zagożdżonka, Poland

The average amount of annual sedimentation in the Staw Górny reservoir is about 1080 m³. When the EROSION-3D and USLE-SDR models were compared, the empirical USLE-SDR model approached the sediments measured in the reservoir. In this case, the EROSION-3D model predicted a 45% lower amount of sediment than was actually measured but offered a satisfactory tool for the identification of the local places endangered by soil water erosion (Fig. 9). The amount of sediments determined by the EROSION-3D model was lower because the EROSION-3D model does not take bottom sediments into account, whereas the amount of sediments quantified by the USLE-SDR model considers bottom sediments as well. The summary of results (modelled and measured) are included in the Table 4. Because of the lowland character of the catchment and because no significant rainfall events occurred, there were no intensive erosion processes. The analyses were done in cooperation with the University of Warsaw under the COST program (CA 16209).

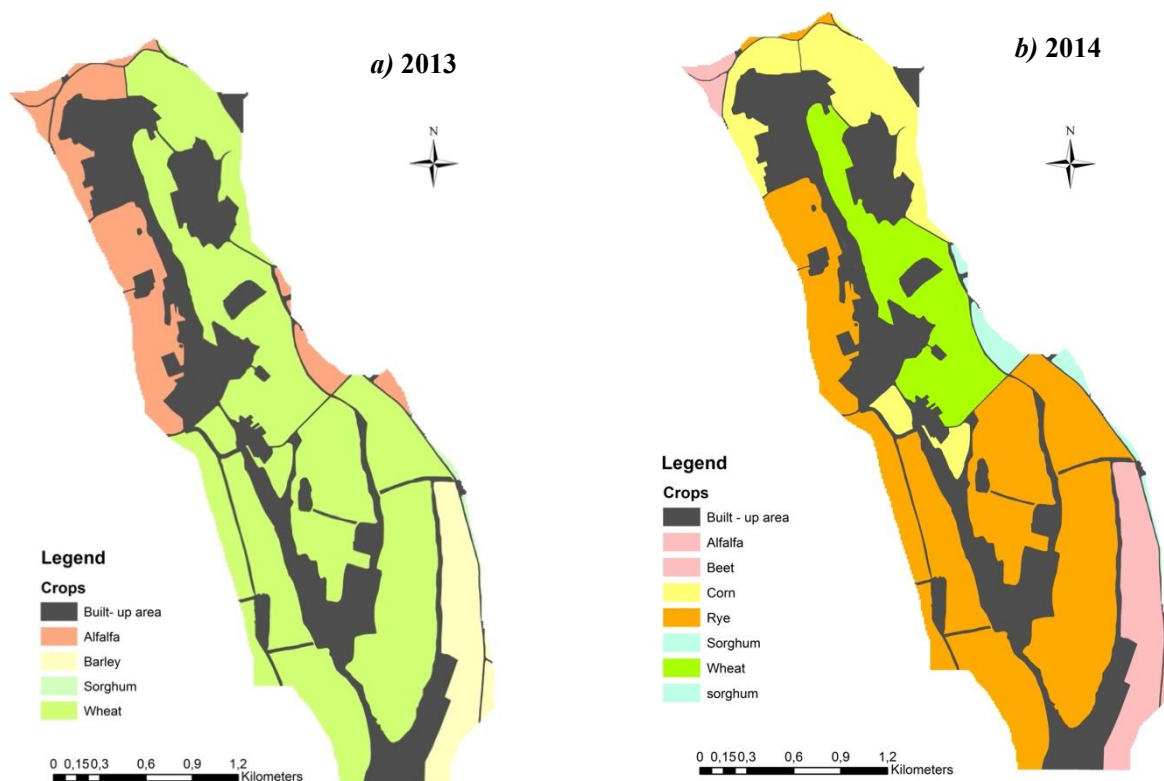


Fig. 3. a) Land-use composition structure, 2013, Svacenický Creek, Slovakia, b) Land-use composition structure, 2014, Svacenický Creek, Slovakia.

Table 3. A summary of the rainfall events and dominant crops during the period selected (2013, 2014)

Year	Rainfall amount [mm]	Predominant crop (>50% of area)
2013	471.1	Winter wheat (<i>Triticum aestivum</i>)
2014	558.28	Rye seed (<i>Secale cereale</i>)

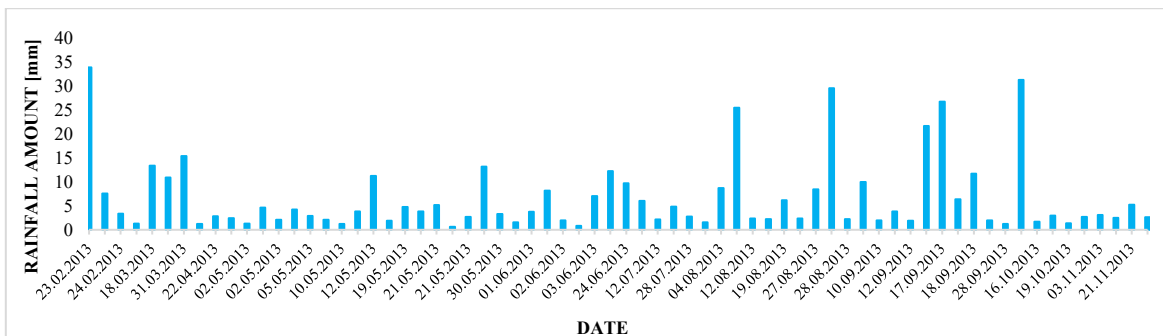


Fig. 4. The amount of rainfall events during the period analysed (February 2013–November 2013).

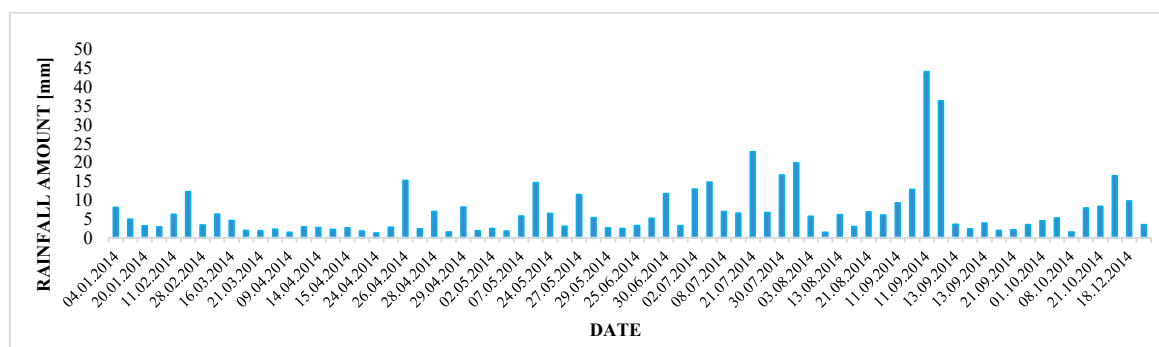


Fig. 5. The amount of rainfall events during the period analysed (January 2014–December 2014).

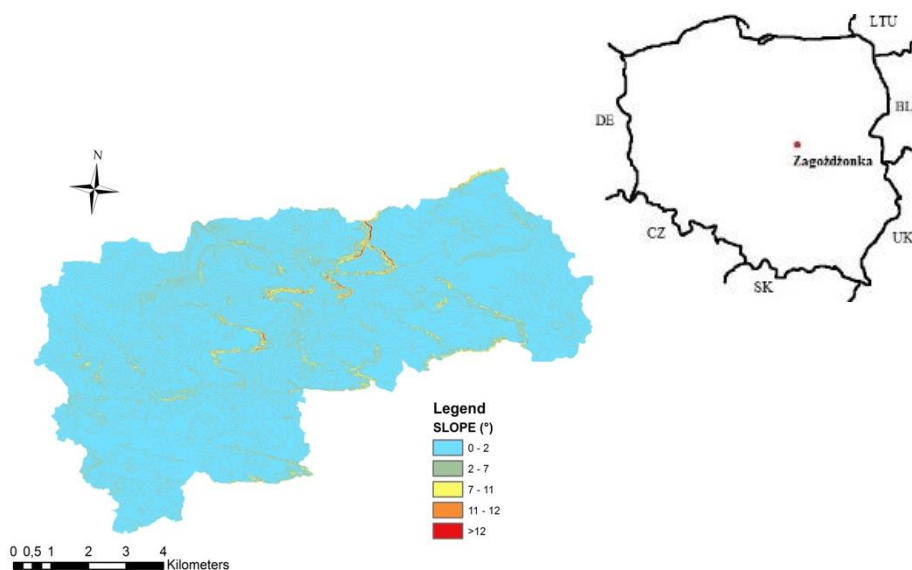


Fig. 6. Localization of the Zagożdżonka research area with a representation of the slopes.

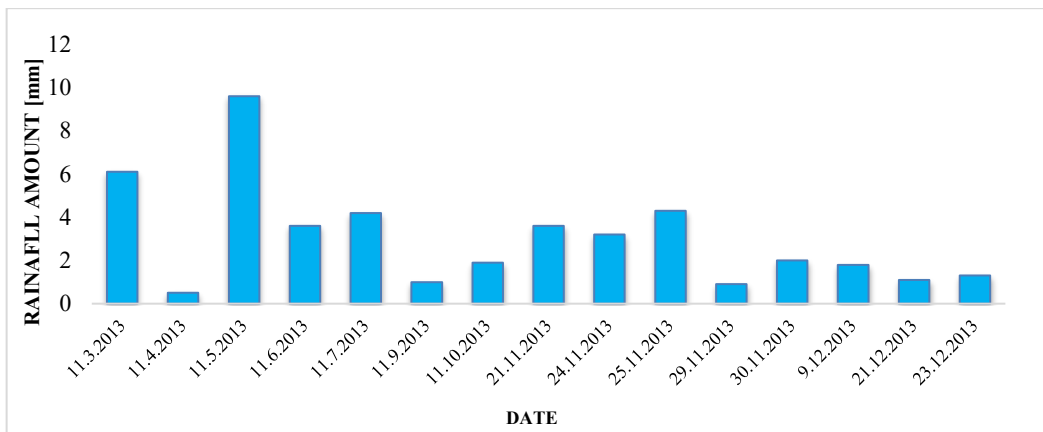


Fig. 7. The amount of rainfall events during the period analysed (November 2013–December 2013).

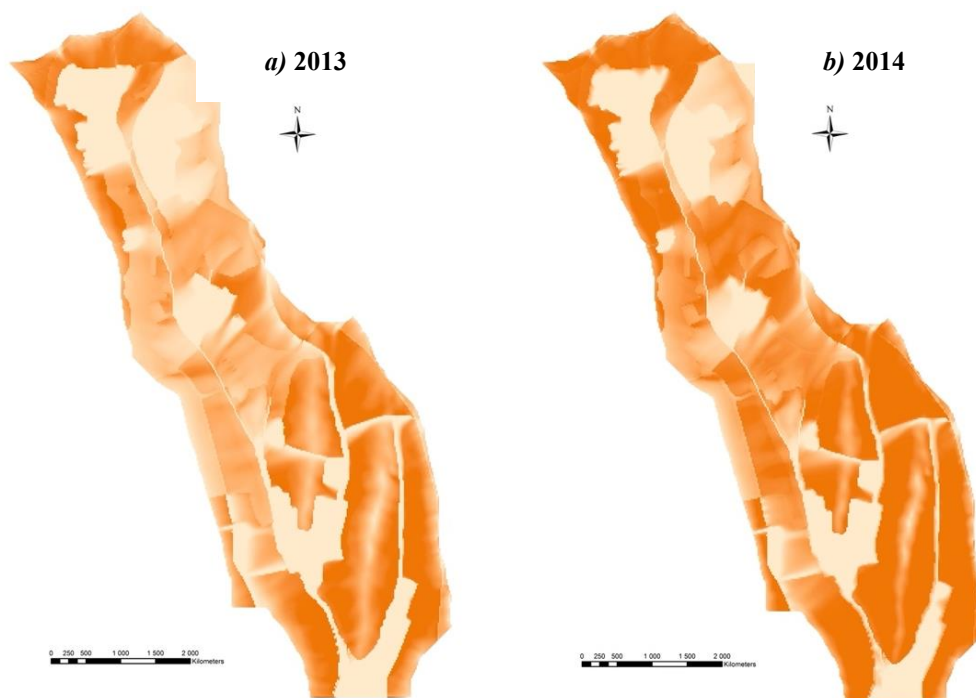


Fig. 8. The modelled results (EROSION-3D model), Svacenický Jarok: a) 2013, b) 2014.

Table 4. Comparison of measured and modelled sediments (USLE-SDR, Staw Górný Reservoir, EROSION-3D model)

Methods for the determination of sediments	Amount of sediments [m ³]
Measurement in the reservoir Staw Górný	1080
USLE-SDR	708
EROSION-3D model	486

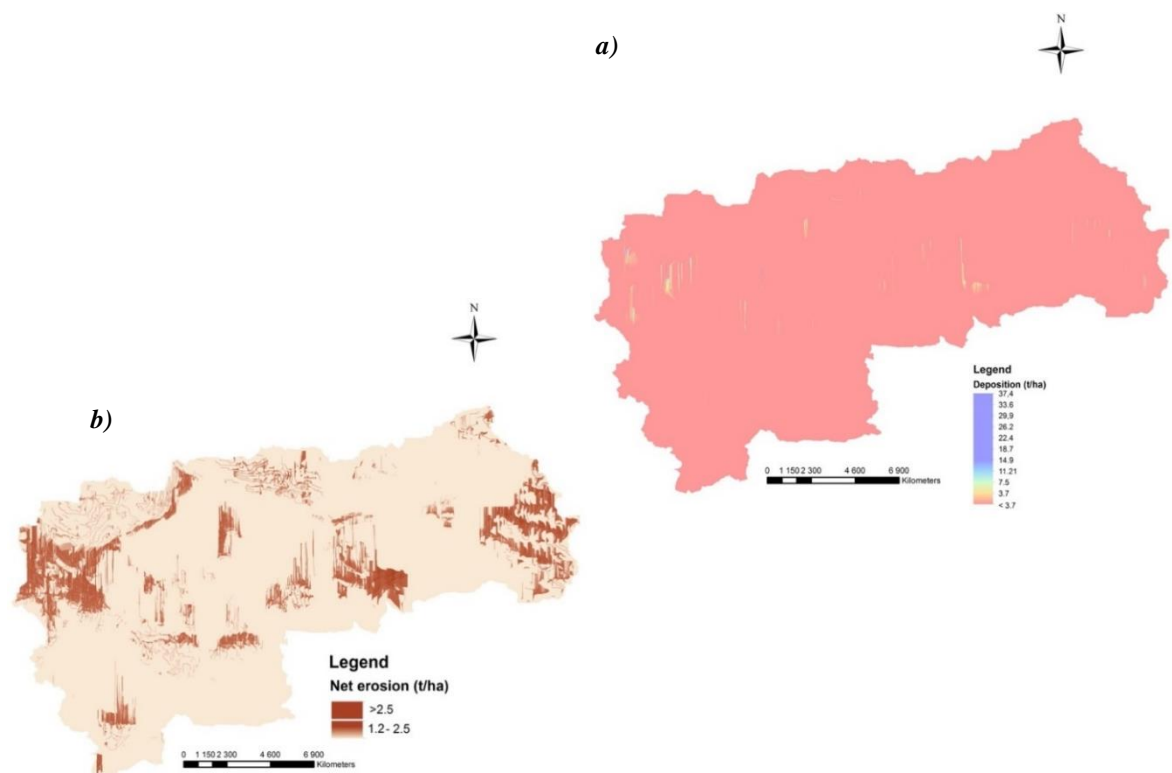


Fig. 9. The amount of deposition (a) and the net erosion (b) during the period evaluated (November 2013–December 2013).

Conclusion

The paper deals with the application and validation of mathematical models used to evaluate the intensity of erosion-transport processes in the Slovak Republic and Poland. The physically-based EROSION-3D model and the empirical USLE-SDR model were used to determine the intensity of the soil erosion and sediment processes. The data modelled were compared with the actual measured data from a bathymetric survey (Slovakia) and from measurements of a water reservoir (Poland).

A large number of mathematical models are available nowadays, but their validation and an appropriate interpretation of the results obtained remain a problem in general. Within the paper, the validation of the models was performed on the basis of the actual measurement of the amount of sediments and on the basis of a continuous series of precipitation events. In the case of the first locality (Svacenický Jarok, Slovak Republic) the EROSION-3D model was validated based on the bathymetric measurement of the sediments in the Svacenický Jarok polder using an Autonomous Underwater Vehicle (AUV) in cooperation with the Institute of Hydrology of the Slovak Academy of Sciences. At the second research site (Zagożdżonka, Poland), the physically-based EROSION-3D model and the empirical USLE-SDR model were used to determine the amount of sediments, and the modelled results were compared with the measured amount of sediments in the Staw Górny reservoir.

Two modelling approaches were used in the study, i.e., a physically-based model and an empirical model; the advantages and disadvantages found within both models were noted. It is not possible to conclude which model can be considered more suitable or better because each of them has certain advantages and disadvantages. In both areas, the EROSION-3D model predicted a lower amount of sediments compared to the amount of sediments observed in the reservoirs. The USLE-SDR empirical model overestimated the impact of precipitation events in comparison with the EROSION-3D model. The advantage of USLE-SDR can be seen in the possibility of considering bottom sediments, while the EROSION-3D model does not include the bottom sediments in the model's calculations. A significant advantage of the EROSION-3D model lies in its ability to determine and analyse different management practices and thus simulate various scenarios of land use changes. Therefore, the practical use of the EROSION-3D model was determined to provide a satisfactory tool for evaluating and locating sites endangered by erosion-sediment processes.

Acknowledgement

This work was supported by the Slovak Research and Development Agency under Contract No. APVV-19-0340, the VEGA Grant Agency No. 1/0632/19 and 1/0782/21. The authors thank the agencies for their research support.

References

- Banasik, K., Mordziński, S. (1982): Wyniki badan i prognoza zamulania malego zbiornika wodnego (Investigative results and prediction of small reservoir siltation). In: *Selected problems of hydraulic structures – designing and construction*. 1982, 62–77. Warsaw Agricultural University Press, Warsaw, Poland.
- Banasik, K., Hejduk, L., Popek, Z. (2005): Sediments problems of small river catchments and reservoir in Poland. Publications of the Institute of Geophysics, PAS, Monograph, 2005, vol. 387, 179–207.
- Banasik, K., Gorski, D., Mitchell, J. K. (2001): Rainfall erosivity for East and Central Poland. In: *Soil Erosion Research for the 21th Century*. 279–282.
- Banasik, K., Skibiński, J., Górski, D. (1995): Methods of estimation of erosion and reservoir sedimentation). In: *Methodology of water resources management in a small river catchments*. Warsaw: Warsaw Agricultural University Press 1995.63–78, 136–143.
- Bennet, H. H. (1933): A Plan for Erosion Control Under the Erosion Section of the Public Works Administration Corporation. File 1-275 Soil Erosion. Central Classified Files: Record Group 48. MD: National Archives and Records Administration 1933.
- Boyle, J. F., Plater, A. J., Mayers, C., Turner, S. D., Stroud, R. W., Weber, J. E. (2011): Land use, soil erosion, and sediment yield at Pinto Lake, California: comparison of a simplified USLE model with the lake sediment record. *Journal of Paleolimnology*, vol. 45, 199–212.
- Dotterweich, M., Stankoviansky, M., Minár, J., Koco, Š., Papčo, P. (2013): Human induced soil erosion and gully system development in the Late Holocene and future perspectives on landscape evolution: The Myjava Hill Land, Slovakia. *Geomorphology*, vol. 201, 227–245. ISSN: 0169-555X.
- García-Ruiz, J. M., Beguería, S., Nadal-Romero, E., González-Hidalgo, J. C., Lana-Renault, N., Sanjuán, Y. (2015): A meta-analysis of soil erosion rates across the world. *Geomorphology*, vol. 239, 160–173.
- Hajigholizadeh, M. M., Hector, A. F. (2018): Erosion and Sediment Transport Modelling in Shallow Waters: A Review on Approaches, Models and Applications. *International Journal of Environmental Research and Public Health*, vol. 15, no. 3, 518.
- Honek, D., Németová, Z., Kohnová, S., Šulc Michalková, M. (2020): Sensitivity analysis of soil parameters and their impact on runoff-erosion processes. *Pollack Periodica*, vol. 15, no. 1, 53–64. ISSN 1788-1994. doi:10.1556/606.2020.15.1.6.
- Iglesias, I., Avilez-Valente, P., Bio, A., Bastos, L. (2019): Modelling the Main Hydrodynamic Patterns in Shallow Water Estuaries: The Minho Case Study. *Water*, vol. 11, 1040; DOI: 10.3390/w11051040.
- Jakeman, A. J., Green, T. R., Beavis, S. G., Zhang, L., Dietrich, C. R., Crapper, P. F. (1999): Modelling upland and instream erosion, sediment and phosphorus transport in a large catchment. *Hydrological Processes*, vol. 13, 745–752.
- Kuznetsov, M. S., Gendugov, V. M., Khalilov, M. S., Ivanuta, A. (1998): An equation of soil detachment by flow. *Soil Tillage Res*, vol. 46, 97–102.
- Markantonis, V., Meyer, V., Schwarze, R. (2012): Valuating the intangible effects of natural hazards – review and analysis of the costing methods. *Natural Hazards and Earth System Sciences*, vol. 12, 1633–1640. DOI: 10.5194/nhess-12-1633-2012.
- Marttila, H., Klöve, B. (2010): Dynamics of erosion and suspended sediment transport from drained peatland forestry. *Journal of Hydrology*, vol. 388, 414–425. DOI: 10.1016/j.jhydrol.2010.05.026.
- Németová, Z., Honek, D., Kohnová, S., Hlavčová, K., Šulc Michalková, M., Sočuvka, V., Velísková, Y. (2020): Validation of the EROSION-3D Model through Measured Bathymetric Sediments. *Water*, vol. 12, no. 4, 1082. <https://doi.org/10.3390/w12041082>.
- Rawat, P. K., Tiwari, P. C., Pant, C. C., Sharama, A. K., Pant, P. D. (2011): Modelling of stream run-off and sediment output for erosion hazard assessment in Lesser Himalaya: need for sustainable land use plan using remote sensing and GIS: a case study. *Natural Hazards*, vol. 59, 1277–1297. DOI: 10.1007/s11069-011-9833-5.
- Renard, K., Foster, G., Weesies, G., McCool, D., Yoder, D. (1997): Predicting Soil Erosion by Water: a Guide to Conservation Plannin with Revised Universal Soil Loss Equation (RUSLE). USDA Agricultural Handbook, vol. 703, 404. ISBN 0-16-048938-5.
- Stankoviansky, M. (2003): Historical evolution of permanent gullies in the Myjava Hill Land, Slovakia. *Catena*, vol. 51, no. 3–4, 223–239. ISSN: 0341-8162.
- Wainwright, J., Parsons, A. J., Michaelides, K., Powell, D. M., Brazier, R. (2003): Linking Short- and Long-Term Soil—Erosion Modelling. In *Long Term Hillslope and Fluvial System Modelling*, 1st ed.; Lang A., Dikau R., Eds.; Springer: Berlin, Heidelberg, 2003; 37–51.
- Wei, H., Nearing, M. A., Stone, J. J., Guertin, D. P., Spaeth, K. E., Pierson, F. B., Nichols, M. H., Moffet, C. A. (2009): A New Splash and Sheet Erosion Equation for Rangelands. *Soil Science Society of America Journal*, vol. 73, 1386–1392. DOI: 10.2136/sssaj2008.0061.

Ing. Zuzana Németová, PhD. (*corresponding author, e-mail: zuzana.nemetova@stuba.sk)
 prof. Ing. Silvia Kohnová, PhD.
 Slovak University of Technology
 Radlinského 11
 810 05 Bratislava
 Slovak Republic