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Empirical models to calculate the snow water equivalent in the high mountain catchments of the Western Carpathians

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Empirical models based on the relationship between snow depth (*SH*) and density (*ρ*) are used to estimate the snow water equivalent (*SWE*) from SH. However, *ρ* is poorly correlated with *SH* while the correlation between *SH* and *SWE* which can be directly obtained from snow measurements, is much better. We derived models based on the *SH-SWE* correlations for two datasets obtained in the high mountain catchments in Slovakia (The Low and Western Tatra Mountains). The models consider time (months from January to April) and elevation zones. Evaluation of the models against independent data showed that they are transferrable to other climatic conditions. About a half of estimated point *SWE* values was well comparable to measured values, i.e. the differences were approximately within ±15%. Substantial overestimation of measured *SWE* by more than 35% was obtained for about 10% of the values in January when the same equation was used for all elevation zones. Our final validation employed independent data from the High Tatra Mountains. It showed that about 60% of *SWE* values calculated for the entire snow courses as an average of 20 values calculated by the derived models from *SH* compared well (±15%) to values obtained by the traditional approach, i. e. as a product of the snow course mean *SH* (20 measurements) and *ρ* (3 measurements). Although the results of our models can be comparable to those provided by models based on snow density, due to recurrent use of SH and almost no correlation between SH and ρ, the models based on the SH-SWE relationship represent in our opinion a more correct approach.

KEY WORDS: regression models, snow cover, hydrology, headwater catchments

Introduction

Snow water equivalent (*SWE)*, i.e. the depth of water that would originate if the snowpack melts, is the most important hydrological characteristic of the snow cover. Point *SWE* values can be obtained from the weight of sampled snow core and area of the snow sampler while *SWE* characterizing greater distances (transects, snow courses) employs also snow density *ρ*. *SWE* is then calculated as a product of the snow course mean snow depth *SH* and mean density *ρ* (calculated from the point *SWE*s and associated *SH*s).

Accurate estimation of *SWE* in catchments is important for correct forecasting of spring runoff, quantification of the groundwater resources replenishment or manipulation of dams located in areas with significant snow accumulation. While the snow cover characteristics at lower elevations of Slovakia in the last decades exhibited decreasing trends (Siman and Slávková, 2019), snow accumulation in the highest mountain catchments, the headwater areas of the most important Slovak rivers, did not yet show such a trend (Holko et al., 2020). However, spatial variability of *SWE* in such catchments is high. Manual *SWE* measurement at snow courses distributed around a catchment remains difficult and time consuming and the automatic measurements (Kinar and Pomeroy, 2015) are expensive and cannot be done at any locations. The remote sensing does not provide a reliable information about the *SWE* variability in smaller mountain catchments (10^0–10^1 km^2). Therefore, emprirical models based on the relationships between snow depth and density taking into account also site altitude, temporal evolution of *ρ* during the winter or snow type, appeared recently to allow the *SWE* estimation from the more easily available SH data (e.g. Jonas et al., 2009; Sturm et al., 2010; McCreight and Small, 2014). Jonas et al. (2009) developed such a model with the aim to facilitate *SWE* monitoring on a catchment scale. It is based on the combination of the look-up tables, regression equations correlating *SH* and *ρ* and residuals between modelled and observed *ρ*. The authors concluded that the model and combination with a few SH measurements characterized a site similarly well as a single SWE measurement. Model validation suggested its robustness to transferability to independent data and even climatic conditions. Sturm et al. (2010) developed a model similar to that of Jonas et al. (2009) and calculated *ρ* taking into account *SH*, day of year and the climate class of snow. They concluded that 90% of the computed *SWE* values fell within ±8 cm of the measured values. McCreight and Small (2014) modified the model of Jonas et al. (2009) for the use daily *SH* data. The complication with daily data is that ρ is small short after the snowfall and gradually increases with snow age due to snow densification. Bohrman et al. (2013) noted that linear snow density-time relationships do not allow for interannual variability in densification rates. They found out that total winter precipitation was the most important driver of snow densification rates while winter mean air temperature and melt-refreeze events were significant locally. Pistocchi (2016) developed an even simpler model that calculated ρ using an initial value at the beginning of the snow cover season and day of year.

Motivation for the development of empirical models estimating *SWE* from *SH* and modelled snow density is to obtain much more *SWE* data for various analyses utilizing the abundant and more easily measurable *SH* data. However, ρ is poorly correlated with *SH* (e.g. Jonas et al., 2009; López-Moreno et al., 2013). Another issue is that in the application of such a model, ρ is first calculated from the (poor) relationship with *SH* and then the same *SH* is used to calculate *SWE* as the product of calculated ρ and *SH*. The method is thus based on the recurrent use of only one independent variable, *SH*, to calculate two unknown variables ρ and *SWE*. Although the final results will not be much different, this could be avoided by calculating *SWE* using regression equations describing the relationships between *SH* and *SWE* which have high correlation coefficients. The equations should employ *SWE* data obtained without considering snow density. Point *SWE* is measured by weighting the snow core sampled by a snow tube. Assuming specific density of water 1000 kg m^{-3}, the snow water equivalent [in mm of water depth] can then be calculated as:

$$SWE = m*(10000/Sa) \tag{1}$$

where

m – is the mass of the collected snow core [kg],

Sa – is the area of the cutting part of the snow tube [cm^2].

SWE based on equation (1) does not need to consider snow density and can be easily derived from the general equation expressing density as a ratio of mass and volume. Various snow tubes and scales have been historically used in different countries (e.g. López-Moreno et al., 2020), but they all work on the same principle, i. e. weighting the snow core collected by the tube. Thus, any measurement employing the snow tube can directly provide *SWE* values. Although *SWE* can be determined directly by equation (1), ρ is used in the snow course measurements as well to obtain the *SWE* for the entire snow course. This is done by multiplication of the mean snow density calculated from a few (3–5) *SWE* measurements by the mean snow depth measured at many more points (10–20) points. Nevertheless, point *SWE* measurements do not need to consider snow density.

The objective of this study is to derive the parameters of equations correlating point values of *SH* and *SWE* and examine the transferability of such empirical models to other areas. The study is based on three datasets obtained in the highest mountains of Slovakia.

Material and methods

To the best of our knowledge, only two datasets containing the long-term *SH* and *SWE* data measured at the snow courses in small mountain catchments exist in Slovakia. The first one (hereafter called the Bystrianka dataset) was measured in the Bystrianka Creek catchment (southern part of the Low Tatra Mountains, catchment area 34.5 km^2, elevations 600–2043 m a.s.l., mean elevation 1200 m a.s.l.) in years 1969–1992. The measurements used in this study were conducted once per month in January, February, March and April. The snow courses were located in different parts of the catchment at elevations 600–2000 m n. m. (Holko, 2000). *SH* was measured by the graduated stick, the snow core was weighted five times at each snow course by the snow tube of Institute of Hydrology SAS which was derived from the US-federal sampler and had the diameter of 3.6 cm (Kozlík, 1967). We used only measurements in which the snow core height was at least 85% of measured *SH* and calculated *SWE* as the product of snow weight in kilograms and number 982.439 (as it follows from equation 1). The dataset contained 2785 values of *SH* and *SWE*. Characteristics of the dataset shown in Fig. 1 confirm poor correlation between *SH* and ρ (calculated from *SWE* and *SH*) and a much better correlation between *SH* and *SWE*.

The second dataset, hereafter called the Jalovčianka dataset, was measured in the Jalovecký Creek catchment (the Western Tatra Mountains, catchment area 22.2 km^2, elevations 820–2178 m a.s.l., mean elevation 1500 m a.s.l.) in years 2012–2022. The measurements were carried out at the end of January, February, March and in April. Methodology of the measurement was similar to that in the Bystrianka Creek catchment. *SH* was measured by the avalanche probe at 20 points and three *SWE* measurements per snow course were obtained with the snow tube Dolfi having the diameter of 8 cm (Hancvencl and Holko, 2019; López-Moreno et al., 2020). *SWE* was calculated by multiplying the snow core weight by 200. The dataset contained 865 values and its characteristics are shown in Fig. 2. Similarly, to the Bystrianka dataset, ρ was poorly correlated with *SH*.

The two datasets were obtained in mountain ranges that are not far from each other. They are divided only by one big mountain valley and the distance between the catchments is approximately 25 km. Yet, meteorological conditions in the catchments during individual winters can sometimes differ. Both catchments are located on the southern side of the mountains, i.e. on their leeward side. However, the Jalovecký Creek catchment is in the first mountain range on the route of moisture bringing air masses from the north and north-western directions (the main direction bringing precipitation to northern Slovakia). Snow cover at some snow courses in the catchment can also be influenced by the windward effects. The Bystrianka catchment is more isolated from the north and north-west. On the other hand, compared to

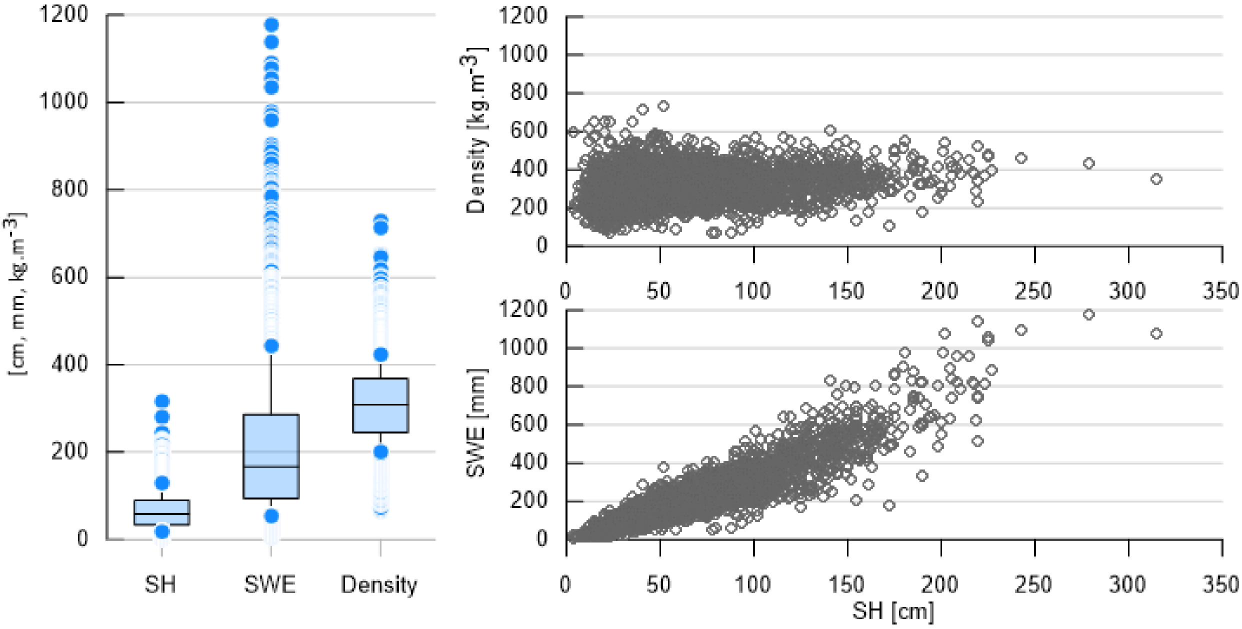

Fig. 1. The Bystrianka dataset (1969–1992); left- boxplots of SH [cm], SWE [mm] and ρ [km m^{-3}]; each boxplot represents 2785 values and shows percentiles 10 and 90 (whiskers), outliers (circles), interquartile ranges and medians (boxes); right – relationships between SH and SWE (calculated by equation 1) and SH and ρ (calculated from point SWE and SH).

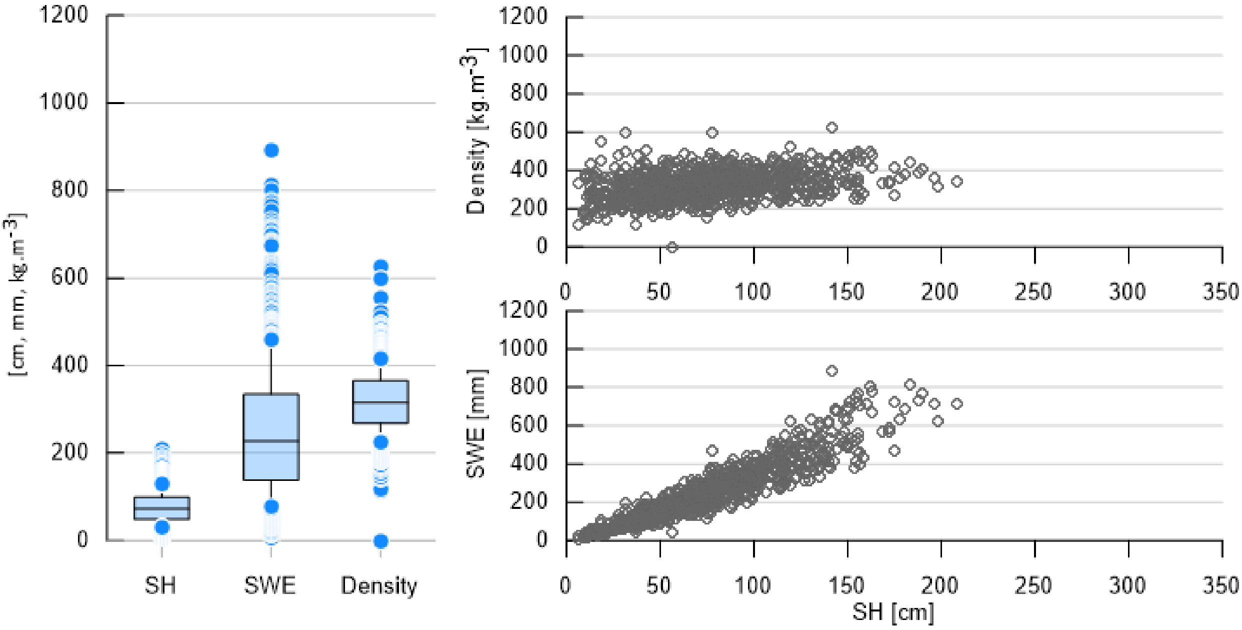

Fig. 2. The Jalovčianka dataset (2012–2022); left- boxplots of SH [cm], SWE [mm] and ρ [km m^{-3}]; each boxplot represents 865 values and shows percentiles 10 and 90 (whiskers), outliers (circles), interquartile range and median (boxes); right – relationships between SH and SWE (calculated by equation 1) and SH and ρ (calculated from point SWE and SH).

the Jalovecký Creek catchment, it can be more often affected by southern weather situations that are important for snow accumulation in some winters.

Slope and intercept of linear regression equations correlating *SH* and *SWE* for January, February, March and April were calculated for each dataset. Division of data into months did not take into account exact dates of measurements, i.e. all the values measured in January, were assigned to that month. In addition to regression equations based on all data from a particular month,

equations for three elevation zones were derived for each month too. The lowest zone (600–850 m a.s.l.) was chosen to represent the snow conditions in the foreland parts of the Slovak high mountain catchments. Zone 850–1400 m a.s.l. should represent the forest zone of mountain catchments and elevation zone 1400–2000 m a.s.l. was considered representative of their highest parts above the tree line.
Equations (empirical models) derived for each dataset were used to calculate SWE for the other dataset that represented independent data collected in different climatic conditions (period 1969–1992 was colder than period 2012–2022). Calculated and measured SWE were compared using the relative bias:

$$Relative\ bias = (SWE_{calculated} - SWE_{measured})*100\ /SWE_{measured}\ [\%] \qquad (2)$$

Basic statistics of the bias were expressed in the boxplots for each month (January to April). Regression plots were used to examine the scatter of measured versus calculated values.
Finally, the performance of the empirical models obtained from the Bystrianka and the Jalovčianka datasets was examined for a smaller set of independent data measured in the neighbouring High Tatra Mountains in springs 2005 to 2007 (February, March, April). The snow courses were located at elevations 1030 to 2080 m a.s.l., their total number was 66 and they were located on the slopes of several mountains and mountain valleys (Patria peak, Gerlachovský štít peak, Slavkovský štít peak and the Velická dolina valley). Three *SWE* values were obtained for each snow course from the snow core sampled by the Dolfi sampler and *SH* measured by an avalanche probe. Relative bias and scatterplots between the measured and calculated *SWE* were used to compare the performance of the models. We also compared the *SWE* obtained for the entire snow course by the traditional approach (multiplication of mean ρ from three *SWE* measurements by the average *SH* from 20 measurements) with the *SWE* calculated as an average of twenty values obtained from *SH* and the empirical models derived from the Bystrianka and the Jalovčianka datasets. The comparison was designed to examine the idea of using the empirical models to provide more *SWE* data for the snow-related analyses.

Results and discussion

Regression analysis confirmed very good correlations between *SH* and *SWE*. Parameters of the obtained linear regression equations for different months and elevation zones are given in Table 1. The relative bias (Figs. 3 and 4) indicates that the performance of equations derived from the Bystrianka dataset (more data, results from a colder climate transferred to the warmer climate) was slightly better that of the models derived from the Jalovčianka dataset (less data, results from a warmer climate transferred to the colder climate). The mean bias (both mean and median) is close to 0%, i. e. measured and calculated values are on average similar, which is the property of correlation equations. However, the interquartile ranges of bias obtained from the Bystrianka dataset are smaller for all months except for April and the upper quartiles are also often smaller than for the Jalovčianka dataset (Table 2).

Table 1. Parameters of linear regression equations correlating point measurements of *SH* and *SWE* (*SWE=Slope x SH+Intercept*) calculated for the Bystrianka and the Jalovčianka datasets; R^2 is coefficient of determination; there was no data for the lowest elevation zone in April in the Jalovčianka dataset; the bold values were calculated for all data in a particular month without considering site elevation

		The Bystrianka dataset				The Jalovčianka dataset			
Month	Elevation	Slope	Intercept	*n*	R^2	Slope	Intercept	*n*	R^2
January	**600–2000**	**3.357**	**-31.225**	**853**	**0.876**	**3.429**	**-39.578**	**216**	**0.891**
	600–899	3.356	-25.413	273	0.860	2.618	-4.606	21	0.949
	900–1399	2.964	-22.133	345	0.829	2.834	-16.813	81	0.948
	1400–2000	3.683	-56.777	235	0.856	3.791	-62.762	114	0.869
February	600–2000	**2.987**	**-11.208**	**870**	**0.862**	**3.366**	**-21.444**	**257**	**0.928**
	600–899	2.667	1.886	412	0.822	2.963	-6.349	30	0.849
	900–1399	2.605	6.092	277	0.629	3.036	-11.755	95	0.956
	1400–2000	3.246	-29.881	181	0.790	3.521	-26.457	132	0.892
March	**600–2000**	**3.659**	**-26.937**	**477**	**0.825**	**4.088**	**-39.771**	**216**	**0.869**
	600–899	3.227	-7.022	69	0.933	3.543	3.430	6	0.963
	900–1399	3.019	1.856	239	0.820	3.542	-19.525	85	0.910
	1400–2000	4.146	-50.518	196	0.788	4.232	-43.328	125	0.832
April	**600–2000**	**3.882**	**-6.148**	**585**	**0.902**	**4.263**	**-40.078**	**176**	**0.856**
	600–899	3.348	-4.866	19	0.767	–	–	–	–
	900–1399	3.548	5.506	275	0.916	3.318	-4.335	60	0.748
	1400–2000	3.930	-0.097	291	0.880	4.323	-27.637	116	0.894

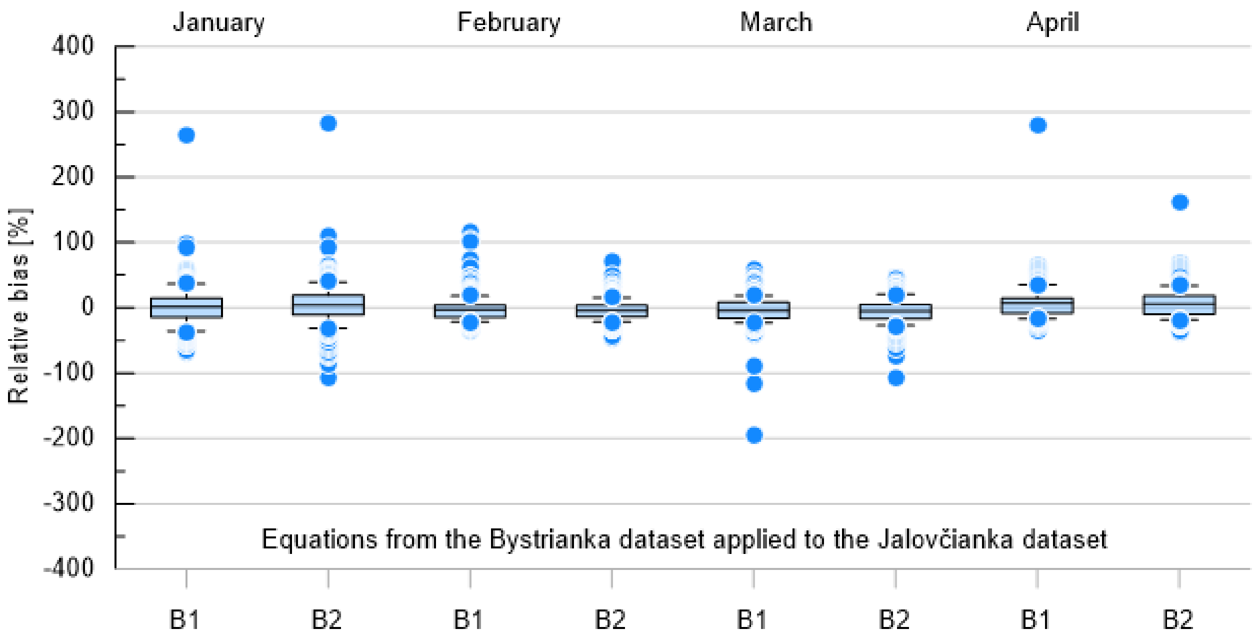

Fig. 3. Relative bias of SWE calculated for the Jalovčianka dataset using empirical models obtained from the Bystrianka dataset; B1 is the bias between measured SWE and SWE calculated by equations considering different elevation zones, B2 is the bias between measured SWE and SWE calculated by the same equation for all elevation zones (the bold numbers in the left part of Table 1); the boxplots show percentiles 10 and 90, outliers, interquartile ranges and arithmetic means.

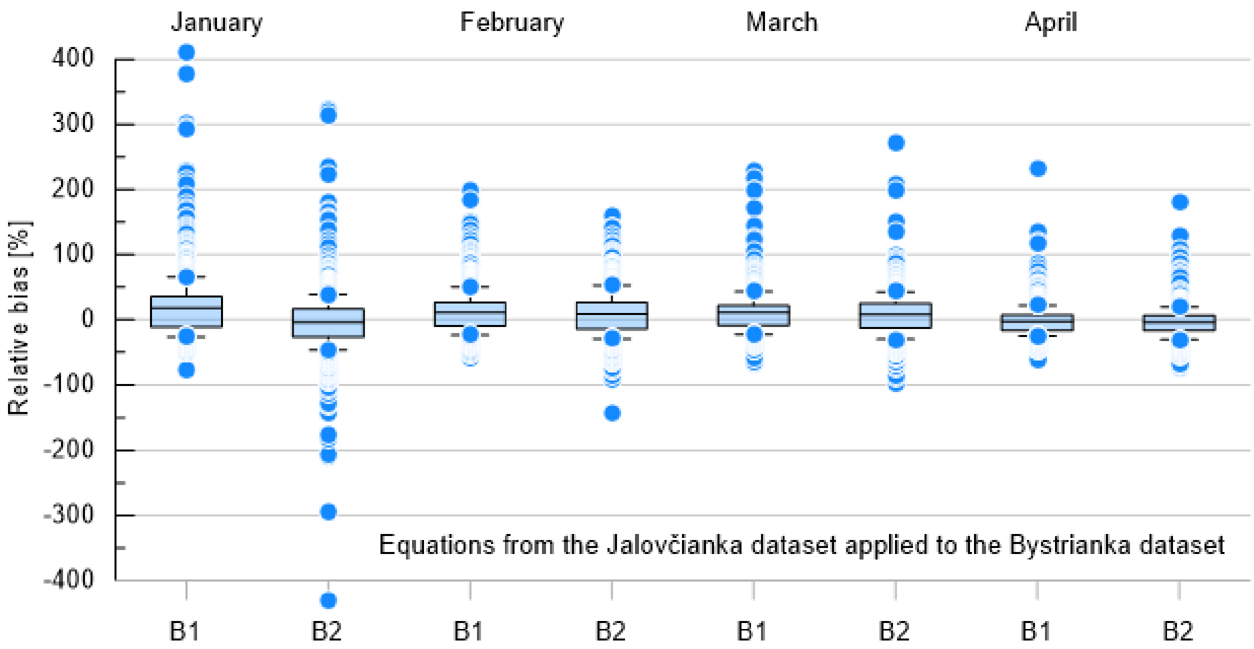

Fig. 4. Relative bias of SWE calculated for the Bystrianka dataset using empirical models obtained from the Jalovčianka dataset; B1 is the bias between measured SWE and SWE calculated by equations considering different elevation zones, B2 is the bias between measured SWE and SWE calculated by the same equation for all elevation zones (the bold numbers in the right part of Table 1); the boxplots show percentiles 10 and 90, outliers, interquartile ranges and arithmetic means.

Table 2 shows that generally the worst results were obtained for the models based on the Jalovčianka dataset for January (the largest upper quartile for B1, the lowest lower quartile for B2 and the largest interquartile ranges for both).
Figs. 3 and 4 and Table 2 provide the information about the accuracy of the SWE estimates. About a half of the estimates can differ from the measured values not more than by 15% which could be acceptable considering the effort needed to measure the SWE in mountains. However, it should be kept in mind that still a substantial number of the estimates can differ much more (see the whiskers and outliers in Figs. 3 and 4). The scatter of estimated SWEs (Fig. 5) is much smaller for the models based on the Bystrianka dataset. However, Fig. 5 also shows that for SWE of approximately 450 mm and more, the calculated values underestimate measured SWE. This phenomenon is less pronounced for the models derived from the Jalovčianka dataset where it becomes visible for the SWE values higher than approximately 600 mm. Fig. 5 shows the results based on the empirical models considering the elevation zones, but similar results were obtained also for the models that did not divide the catchments into elevation zones.
Greater negative intercepts in Table 1 mean that negative values of SWE can be calculated for very small SH.
Evaluation of the models obtained from both datasets against the data measured in the High Tatra Mountains showed that the differences between the models were very small (Fig. 6), although the models derived from the Jalovčianka dataset on average overestimated measured values slightly more. On the other hand, the snow course SWE values calculated with the models derived from the Jalovčianka dataset compare to values obtained by the traditional approach slightly better than those calculated with the models derived from the Bystrianka dataset (Fig. 7). The differences of 80% of calculated snow-course SWE valus (by models from the Jalovčianka dataset) were in the interval from -15% to +33%. Analogical differences for the models calculated from the Bystrianka dataset were within the interval from -25% to +24%. López-Moreno et al. (2020) concluded that the uncertainty in snow density measurements conducted by different snow tubes is approximately within 10–15%. Approximately 60% of the snow course SWE values estimated by the empirical

Table 2. Values of the lower and upper quartiles (P25, P75) and the interquartile ranges of SWE relative bias [%] when the equations from the Bystrianka dataset are applied to the Jalovčianka dataset (The Bystrianka dataset) and vice versa (The Jalovčianka dataset)

		The Bystrianka dataset			The Jalovčianka dataset		
		P25 [%]	P75 [%]	Interquartile range [%]	P25 [%]	P75 [%]	Interquartile range [%]
January	B1	-15	15	30	-11	35	46
	B2	-11	18	29	-26	17	43
February	B1	-15	4	19	-10	27	37
	B2	-14	4	18	-14	27	41
March	B1	-16	8	24	-9	22	31
	B2	-17	5	22	-12	25	37
April	B1	-9	15	24	-16	7	23
	B2	-10	18	28	-17	7	24

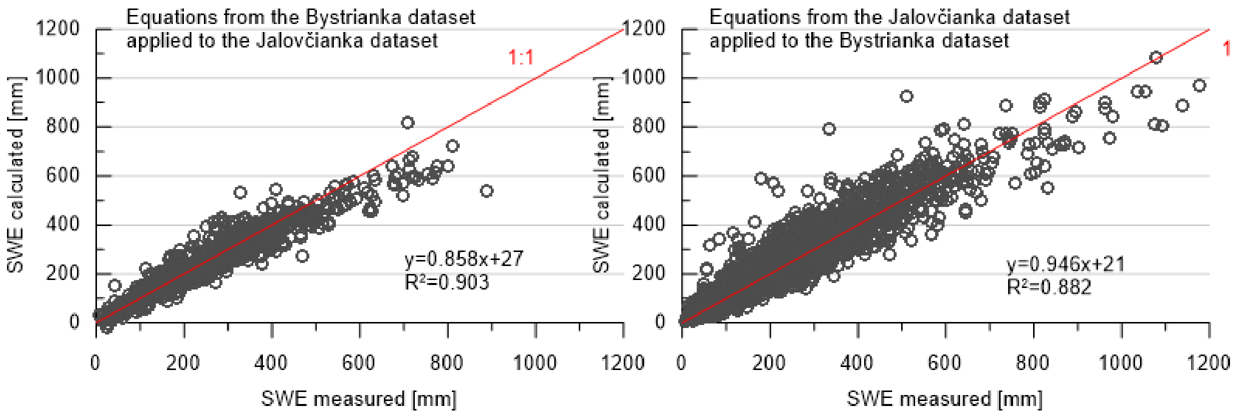

Fig. 5. Calculated versus measured SWE and parameters of linear regression equations describing the relationships between measured and calculated values; the calculated SWE values are based on the models considering elevation zones.

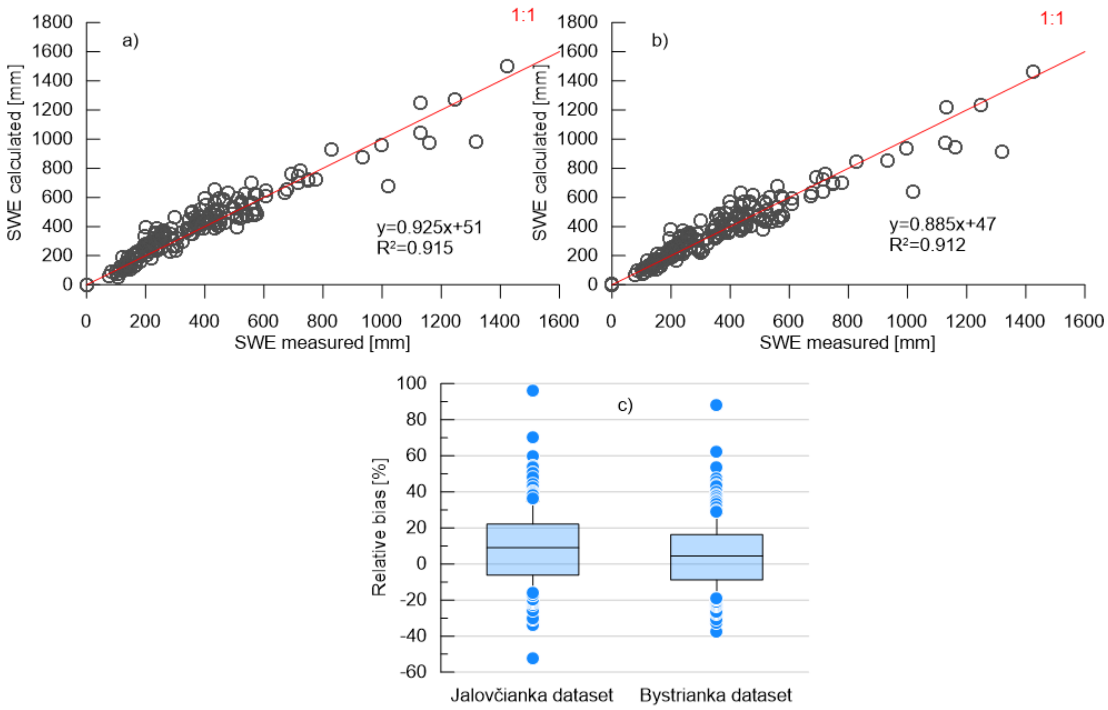

Fig. 6. Comparison of measured <u>point SWE</u> in the High Tatra Mountains and SWE calculated by the models derived from the Jalovčianka dataset (a) and from the Bystrianka dataset (b); the models took into account elevation of the sites; c) relative SWE bias, the boxplots show percentiles 10 and 90, outliers, interquartile ranges and arithmetic means.

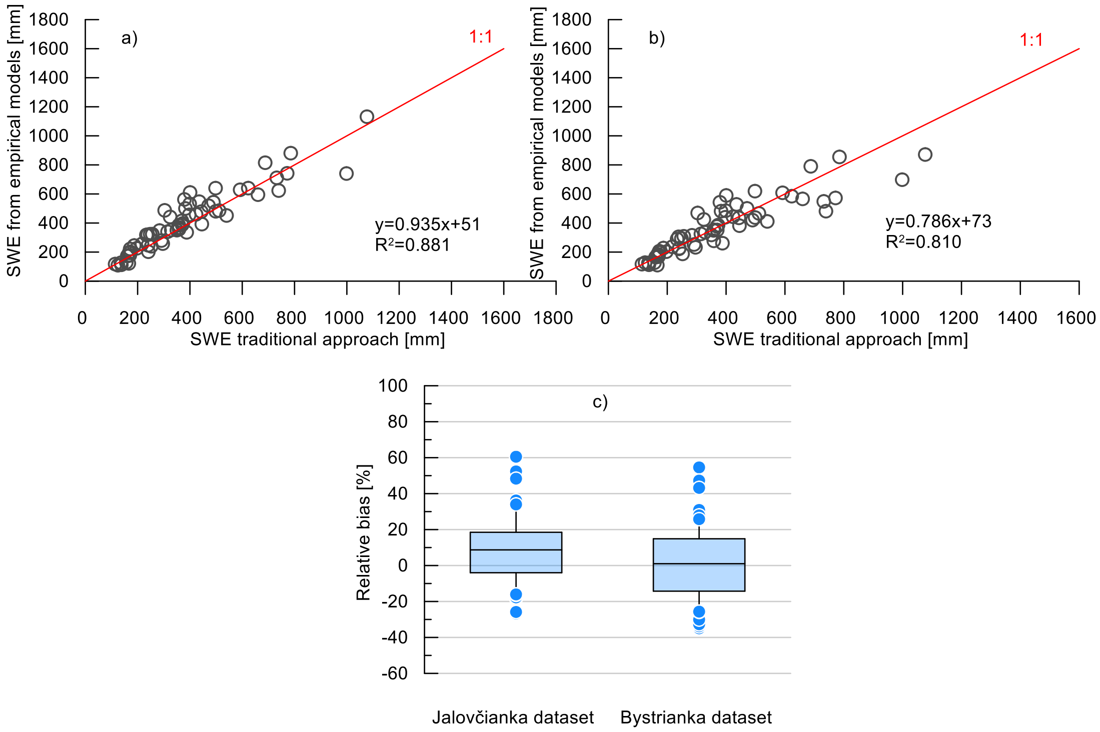

Fig. 7. Comparison of the SWE <u>for the entire snow courses</u> in the High Tatra Mountains calculated by the traditional approach (as a product of snow course mean SH and ρ based on measurements) and as the average of 20 SWE values calculated by empirical models derived from (a) the Jalovčianka dataset and (b) from the Bystrianka dataset that took elevation zones into account; c) relative SWE bias, the boxplots show percentiles 10 and 90, outliers, interquartile ranges and arithmetic means.

models differed from the measured values obtained by the traditional approach not more than by 15%. While this could be quite a good result for the simple empirical models, it also means that the accuracy of quite many SWE estimates (40%) can be substantially worse than the systematic errors in measured data. Users of such models should therefore carefully consider their application.

Conclusion

We proposed alternative empirical models to estimate the *SWE* from *SH* measurements based on the correlations of *SH* and *SWE*. The validation indicates that the models which we believe are more correct than earlier models based on the correlations between *SH* and ρ, could be useful to obtain more *SWE* data for some snow-related analyses.

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