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Comparison of saturation models in complex hillslopes

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Hillslopes of natural catchment have a complex geometry. In complex hillslopes, combining different cases of plan shape (convergent, parallel and divergent) and profile curvature (concave, convex and straight) nine different geometries are created. In prediction of the surface and subsurface runoff of catchments based on saturation excess runoff mechanism, the saturated and unsaturated zones of hillslopes must be first separated. Subsurface travel time of hillslope is dependent on saturation attributes. In this research, a new saturation model, called Gamma, was developed to predict the saturated zone length and subsurface travel time in complex hillslopes. An analytical formula was introduced to calculate saturation zone length in Gamma model. Results of Gamma model, namely the saturation zone length and subsurface travel time, were compared with the results given by two other complex saturation models W and Sigma. The results of the three models were relatively close to each other in convergent and parallel hillslopes of different profile curvature type. However, due to the existence of an analytical equation for estimation of saturated zone length in the Gamma model, this model is recommended. It should be noted that for straight divergent and convex divergent hillslopes, the Gamma model is not suitable and Sigma or W model should be used.

KEY WORDS: saturation excess runoff mechanism, complex hillslope, subsurface travel time

Introduction

Investigating cathments‘ saturation and runoff production in hydrology is crucial for many applications. For instance, runoff formation is linked to flood mapping (Szolgay et al., 2003). Infiltration within soil generates the groundwater recharge (Aldarir et al., 2022). The presence of water in soils affects soil hydraulic properties, e.g. the soil retention curves (Kandra et al., 2015).

The infiltration (Hortonian) and saturation (Dunne-Black) excess are two of the main runoff formation mechanisms in catchments. In the latter, the subsurface flow produces saturated zone in downstream of the hillslope (Dunne and Black, 1970 a, b). These two runoff formation mechanisms can be found in a catchment at the same time. In hilly catchments with vegetation cover which have steep hillslopes, large part of the observed runoff usually comes from subsurface runoff (Hewlett and Hibbert, 1963; 1967; Anderson and Burt, 1978).

Due to their complex nature, separation of surface and subsurface flow in catchments is difficult and an accurate technique has not been provided for this purpose yet.

Many rainfall-runoff models used for simulation of catchment are based on the infiltration excess mechanism. Catchment surface becomes saturated in such models from upstream and the entire surface of a hillslope is usually assumed to contribute to runoff. Separation of saturated and unsaturated areas in hillslopes during rainfall is necessary to account for the infiltration excess mechanism of runoff formation. Estimation of these areas considering the temporal variability of rainfall intensity is more difficult.

The dynamic of interaction between saturated and unsaturated zones has been examined by several researchers using numerical simulations (Freeze and Harlan, 1969; Freeze, 1971;1972 a, b; Beven, 1982).

Subsurface flow and saturation of hillslopes are affected by many parameters such as soil specifications (porosity, soil hydraulic conductivity and soil thickness), recharge rate, vegetation cover, the geometry of hillslopes.

Hillslopes have different geometries. In terms of profile curvatures, hillslopes are divided into three cases which are concave, convex and straight, and, regarding the plan shape, they are grouped into convergent, parallel and divergent. Generally nine different geometries are considered for hillslopes which are called complex hillslopes. According to past investigations, this geometry can have a significant effect on the surface flow (Singh and Agiralioglu, 1981a, b; 1982; Noroozpour et al., 2014; Sabzevari and Noroozpour, 2014) and subsurface flow (O'Loughlin, 1981; Aryal et al., 2005; Hilberts et al., 2004; 2007; Berne et al., 2005; Troch et al., 2002; 2004; Sabzevari and Noroozpour, 2014).

O'Loughlin (1981) derived criteria for the existence of saturated area on draining hillslopes in natural catchments. He showed that the size of saturated zone on undulating hillslopes with gradational soils strongly depends on topographic convergence or divergence. Takasao and Shiiba (1981) concluded that the effect of flow concentration on hydrological responses varies in convergent and divergent hillslopes, and with the magnitude of rainfall. They also noted that convergent hillslopes with deep soils had delayed runoff responses, whereas hillslopes with shallow soils produced highly peaked hydrographs. The analytical solutions to a kinematic wave equation which are expressed in terms of storage using the mapping method of Fan and Bras (1998) were evaluated through applying them to nine hillslopes with different plan shapes and profile curvatures by Troch et al. (2002). Troch et al. (2002) also found that dynamic response of the hillslopes is strongly dependent on plan shape and bedrock slope. It was concluded that convergent hillslopes tend to drain much more slowly than divergent ones due to a reduced flow domain near the outlet. Aryal et al. (2005) derived relationships describing response times for landscape saturation and subsurface flow for idealized hillslopes after a change happened in water balance in terms of similarity parameters given by their topographic, soil and climatic attributes. They investigated the effects of geometry of hillslopes on the amount of saturation in hillslopes, and then, by calculating the length of saturation zone (SZL) in steady rainfall condition, they estimated travel time of subsurface flow. According to their derived equations, each factor affecting the amount of saturation in hillslopes can influence the subsurface travel time (STT) too. Their results showed that the STT in divergent hillslopes is twice that of convergent hillslopes, and concave slopes tend to have lower travel times than planar or convex slopes. Sabzevari et al. (2010) determined the SZL of complex hillslopes based on the Sigma saturation index previously given by Troch et al. (2002). In their study, an analytical equation was presented according to the saturation model of complex hillslopes to calculate the travel time of subsurface flow. They related the saturation capacity, and subsurface travel time of complex hillslopes to soil parameters (thickness, porosity and hydraulic conductivity), geometric characteristics of hillslope (length, plane shape and profile curvature) and rainfall recharge rate. Based on their results, the convex hillslopes show smaller SZL than the concave hillslopes and take greater STT compared with straight and concave ones, whereas in convex-convergent hillslopes, due to their convergence, there is much inclination towards saturation. On average, the STT in divergent hillslopes is twice that for convergent hillslopes. The concave hillslopes saturate sooner in comparison with the straight and convex ones, with shorter STT. Sabzevari and Noroozpour (2014) introduced an instantaneous unit hydrograph model for simulating runoff hydrographs for complex hillslopes. The model is able to estimate surface and subsurface flows in catchments based on the saturation-excess mechanism. The model was used to predict the direct runoff hydrograph (DRH) and subsurface flow hydrograph in Walnut Gulch No. 125 catchment in Arizona, USA. Their results showed that the geometry can change the peak of DRH.

In this research, a saturation model, called Gamma model hereafter, is introduced which considers a simpler geometry in comparison with Sigma saturation model. The results of Gamma model were compared with those of saturation model results given by Sabzevari et al. (2010) and Aryal et al. (2005). The main goals of this research are to (1) provide a saturation model for complex hillslopes with a simpler geometry and to calculate subsurface travel time, and (2) compare the results of saturation models of Sigma and Gamma with that of W saturation model presented by Aryal et al. (2005).

Sigma model description

Evans (1980) provided a topography model to produce complex hillslopes as follows:

$$z(x, y) = E + H(1 - x / L)^n + \omega y^2 \tag{1}$$

where z is the elevation, x is the horizontal distance lengthwise measured in the downstream length direction of the surface, y is the horizontal distance from the longitudinal axis in the direction perpendicular to the length direction (the width direction), E is the minimum elevation of the surface above an arbitrary datum, H is the maximum elevation difference defined by the surface, L is the horizontal length of the surface, n is the profile curvature parameter, and is the plan shape parameter.

Fig. 1 illustrates a convergent convex hillslope with soil thickness of D on the bedrock which has the same curvature as the soil surface.

This hillslope is recharged by a rainfall, and due to infiltration of water into the soil, soil moisture profile is formed. Soil moisture storage is related to soil characteristics, recharge rate, and geometry of hillslope (Troch et al., 2002). The width of the complex hillslopes is calculated from the following equation (Talebi et al., 2008):

$$W(x) = c_w \exp\left\{ \frac{2\omega L^2}{n(2-n)H} \left(1 - \frac{x}{L}\right)^{2-n} \right\} \tag{2}$$

where c_w defines the hillslope width at $x=L$. It is supposed that the soil thickness is the same throughout the hillslope.

The steady-state relative saturation function in Sigma model is given by Troch et al. (2002) and Talebi et al. (2008):

$$\sigma(x) = \frac{S(x)}{S_c(x)} = \frac{\left[\frac{fLN}{nkH} (1 - \frac{x}{L})^{1-n} A(x) \right]}{\left[W(x) D(x) f \right]} \tag{3}$$

$$A(x) = \int_0^x W(u) du \tag{4}$$

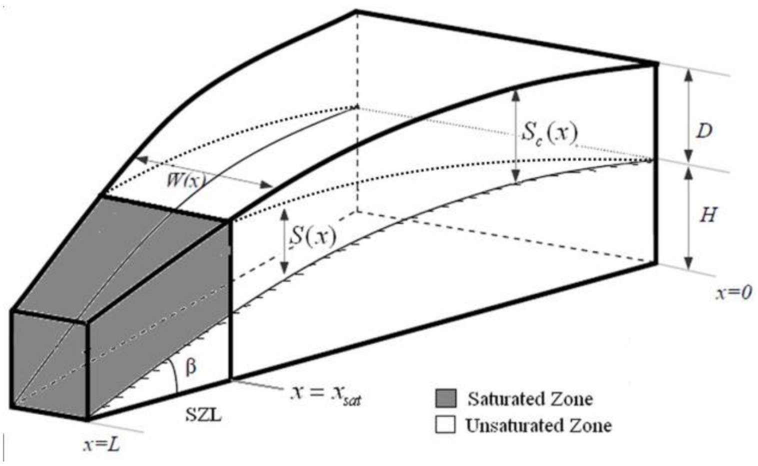

Fig. 1. Schematic of convergent convex hillslope.

where *S(x)* is the soil moisture storage, $S_c(x)$ is the soil moisture storage capacity, *A(x)* is the drainage area at the distance *x*, *N* is the recharge rate to the saturated layer, *k* is the saturated soil hydraulic conductivity, *f* is the drainable porosity, and *D(x)* is the average soil thickness.

According to Fig. 1, any point of the hillslope with the moisture storage equal to the moisture storage capacity σ=1, belongs to the saturation zone (Sabzevari et al., 2010). By solving numerically of the equation [$\sigma(x)-1=0$], the SZL can be obtained.

In Eq. (4), except for *n*=1, the function of A(x) does not have an analytical solution and could only be solved numerically.

Subsurface flow travel time is a key parameter in some rainfall-runoff models to estimate the subsurface flow of catchment. In most catchments with high soil permeability, the contribution of subsurface flow to DRH is significant. Sabzevari et al. (2010) offered the Eq. (5) for calculation of SST in complex hillslopes based on the Sigma saturation model and Darcy equation:

$$T = \frac{L\ f\left[(1-\frac{x_{sat}}{L})^{2-n}-1\right]}{nk\bar{S}(n-2)} \tag{5}$$

where x_{sat} is the coordination of the saturated zone boundary. According to Eq. (5), the coordination of the saturated zone boundary in each hillslope is a key parameter in calculating STT.

W model description

Aryal et al. (2005) used the geometric equations of Zaslavsky and Rogowski (1969) for modelling of complex hillslopes. Eq. (6) describes the geometric profile of concave and convex hillslopes:

$$\begin{aligned} z &= H[\frac{\tan^{-1}\{B(X-1)\}}{\tan^{-1}(B)}+1] \qquad Concave \quad B>0 \\ z &= H[\frac{\tan^{-1}\{BX\}}{\tan^{-1}(B)}] \qquad Convex \quad B<0 \end{aligned} \tag{6}$$

where *B* is the curvature parameter and *X=1-(x/L)* is the normalized distance.

Aryal et al. (2005) obtained the value of saturation zone boundary by solving the following equation:

$$\frac{X_s(2-X_s+X_sCR)}{1+CR} = 1-\frac{fDk\bar{s}}{NL}\frac{f(X_s)}{\lambda(X_s)} \tag{7}$$

$$\lambda(X_s) = (1+CR)/[2(1-X_s+X_sCR)]$$

where *CR* signifies the convergence degree of hillslope (*CR*>1 for convergent, *CR*<1 for divergent, and *CR*=1 for parallel hillslope) which is the ratio of upstream width to the width of hillslope at the outlet, $X_s=1-(x_{sat}/L)$ is the normalized length of saturation zone, $\lambda(x_s)$ is the plan shape function, $f(X_s)=dz/dx$ is the curvature profile function.

Aryal et al. (2005) presented the Eqs. (8) and (9) for calculation of STT in concave and convex hillslopes, respectively:

$$T = \frac{fL}{k\bar{s}}\tan^{-1}(B)[\frac{3+B^2}{3B}-(\frac{BX_s^{\ 3}}{3}-BX_s^{\ 2}+BX_s+\frac{X_s}{B})] \tag{8}$$

$$T = \frac{fL}{k\bar{s}}\tan^{-1}(B)[\frac{3+B^2}{3B}-(\frac{BX_s^{\ 3}}{3}+\frac{X_s}{B})] \tag{9}$$

As the value of *B* approaches 0 (a straight hillslope), Eq. (8) reduces to $T=fL(1-X_s)/(ks)$.

Gamma model development

Gamma model introduced in this study is based on the topography model already used by Norbiato and Borga (2008) and Berne et al. (2005). Due to the dependence of the width function derived from Evans (1980) on both plan shape parameter (*n*) and profile curvature parameter (ω), the resultant drainage area equation (*A(x)*) can not be solved analytically for any *n*. This limitation is eliminated in Gamma model by assuming an exponential form for width function as:

$$W(x) = c\exp(ax) \tag{10}$$

where c corresponds to the surface width at the upstream ($W(x=0)$) and a is a shape parameter quantifying the degree of convergence ($a < 0$) or divergence ($a > 0$) of the hillslope. The hillslope drainage area function is obtained analytically by integrating Eq. (10) as:

$$A(x) = \frac{c}{a}[\exp(ax) - 1] \tag{11}$$

For investigation on subsurface flow of hillslopes based on Fan and Bras (1998) equation, Norbiato and Borga (2008) considered the equation of profile curvature in complex hillslopes as:

$$z(x) = \alpha + \beta x + \gamma x^2 \tag{12}$$

In Eq. (12), values of β are always negative, positive values of γ define concave profiles, negative values of γ produce convex profiles, and for $\gamma = 0$ the profile is straight. Comparing Eq. (12) and Eq. (1) for n=2, the relation between the parameters of two equations would be: $\alpha = E + H, \beta = -2H / L, \gamma = H / L^2$

For the surfaces defined by Eq. (12), the local slope at each point of hillslope, $s*$, is obtained from:

$$s^* = \frac{dz}{dx} = \beta + 2\gamma x \tag{13}$$

The subsurface flow rates can be described with a kinematic wave approximation of Darcy's law as (Troch et al., 2002):

$$Q = -k\frac{S(x)}{f}\frac{dz}{dx} \tag{14}$$

According to Fig. 1, any point of the hillslope with the storage equal to the storage capacity [$S(x)=S_c(x)$], belongs to the saturation zone. The steady-state subsurface flow rate would be $Q(x) = NA(x)$. According to the Eqs. (14) and (13), it could be written:

$$S(x_{sat}) = \frac{-NA(x_{sat})f}{k(\beta + 2\gamma x_{sat})} \tag{15}$$

Computing the boundary of saturation zone, the value $S(x)=S_c(x)=w(x_{sat})Df$ is considered. Therefore:

$$\frac{-NA(x_{sat})}{k(\beta + 2\gamma x_{sat})} = w(x_{sat})D \tag{16}$$

Replacing the Eqs. (10) and (11) into Eq. (16) and solving the final equation for the boundary of saturation zone yields:

$$x_{sat} = \frac{2lambertw\left\{\frac{N}{2Dk\gamma}\exp(\frac{N + Dak\beta}{2Dk\gamma})\right\} \times Dk\gamma - N - Dak\beta}{2Dak\gamma} \tag{17}$$

where *lambertw(x)* evaluates Lambert's w function at the elements of x, a numeric matrix or a symbolic matrix. Lambert's w solves the equation $w\,exp(w)=x$. So the SZL is obtained from the relation: $L_s=L-x_{sat}$.

Eq. (17) is an analytical equation of saturation zone boundary in Gamma model. Based on Gamma model, the STT can be computed more simply.

According to Darcy equation for porous media, the velocity of water into soil is calculated as:

$$v = \frac{k\,s^*}{f} = \frac{dx}{dt} \tag{18}$$

By substituting the value of $s*$ from Eq. (13) in Eq. (18) and integrating with bounds t = 0 to T, and x = 0 to x_{sat} for travel time of the unsaturated zone, we obtain:

$$T = \frac{-f}{2k\gamma}[\ln(\frac{\beta + 2\gamma x_{sat}}{\beta})] \tag{19}$$

Replacing x_{sat} from Eq. (17) into Eq. (19), the STT of the complex hillslopes can be computed.

Models application

To investigate the effect of geometry on saturation of complex hillslopes, the nine basic complex hillslopes considered by Sabzevari et al. (2010) are studied to investigate STT of complex hillslopes. Fig. 2 illustrates the nine basic hillslope types that are formed by combining three plan shapes and three profile curvatures. The geometrical parameters of Sigma model for the nine characterized hillslopes are listed in Table 1. The length of all hillslopes is 100 m, the maximum elevation difference defined by the surface is 26.8 m, the average slope is 0.27, and the maximum value for y is 25 m. The curvature shape factor for hillslopes varies between 0.5 and 1.5 and the plan shape factor between $\omega = -H / L^2$ (for divergent hillslope) and $\omega = +H / L^2$ (for convergent hillslope) (Troch et al., 2002; Talebi et al., 2008).

The straight hillslopes have profile curvature parameter of unity, while the plan shape factor for parallel hillslopes is zero. The geometric parameters of hillslope in natural catchment usually are calculated by using geographic international system (GIS) technique.

To make the results of the models W, Gamma, and Sigma, comparable, the geometric parameters of hillslopes as regards plan shape and curvature profile of the defined study scheme should be equivalent to each other. For this purpose, geometrical parameters of Table 1 should be changed to geometries of the Gamma and W models. To do this, hillslope area, length, average slope, and the upstream width in all three geometries are considered the same for nine cases and geometric parameters are calculated. For instance, Eqs. (1), (6), and (12) are curvature equations for three geometries so by comparing the results of the two latter with the former, curvature parameters B and γ could be evaluated.

Comparing curvature and plan shape equations, and their solutions for different values of n and ω, the parameters

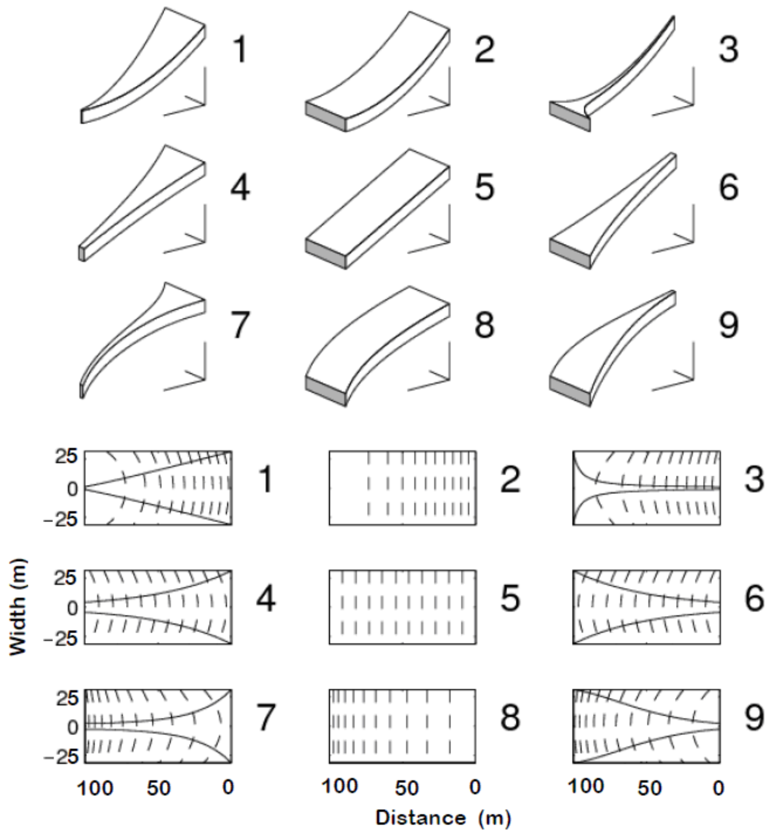

Fig. 2. A three-dimensional view (top) and a two-dimensional plot of the contour lines and slope divide the bottom of the nine hillslopes considered in this study (After Hilbert et al., 2004).

Table 1. Geometrical parameters for nine complex hillslopes (Sabzevari et al., 2010)

Hillslope Number	Profile Curvature	Plan Shape	*n*	$\omega [10^{-3} m^{-1}]$
1	Concave	Convergent	1.5	+2.7
2	Concave	Parallel	1.5	0
3	Concave	Divergent	1.5	-2.7
4	Straight	Convergent	1	+2.7
5	Straight	Parallel	1	0
6	Straight	Divergent	1	-2.7
7	Convex	Convergent	0.5	+2.7
8	Convex	Parallel	0.5	0
9	Convex	Divergent	0.5	-2.7

of models W and Gamma would be obtained as shown in Table 2. In model W, the convergence coefficient CR of complex hillslopes, is derived as:

$$CR = W(0)/c_w = \exp\left\{\frac{2\omega L^2}{n(2-n)H}\right\} \tag{20}$$

Considering equation $\omega=-H/L^2$, for divergent hillslopes, the CR coefficient would be CR=exp[-2/(n(2-n)] , and for convergent hillslopes with $\omega=H/L^2$ it would be CR=exp[-2/(n(2-n)].

Results and discussion

In this section, the results of response of subsurface flow in complex hillslopes are discussed. Saturation rate and subsurface travel time of different hillslopes are compared according to the three models. Since choosing one model as more factual requires laboratory researches, in this study W model is taken as basis and the results of the other two models are compared with those of W model. The method of RMSE is used to compare the results. The value of RMSE is calculated from:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{m}(x_C - x_w)^2}{m}} \qquad (21)$$

where x_c is the values from the Gamma or Sigma models, and x_w is the values from W model. The recharge rates into soil layer were considered so that the saturation level is observed at downstream of hillslopes. Since some hillslopes saturate too late (Sabzevari et al., 2010), the rainfall intensity should be assumed high enough. Figs. 3 and 4 show the values of saturated zone lenght (SZL) and subsurface travel time (SST) in complex hillslopes for different models. Table 3 presents the values of RMSE of two models W and Sigma.

To simulate the responses of convex-convergent hillslopes (Figs. 3g, 4g), recharge rates in the range of 35 to 60 mm day^{-1} were utilized because this hillslope saturates very much. For convex-convergent hillslope, The RMSE values for models of Gamma and Sigma are, respectively 32.1 mm day^{-1} and 31.3 mm day^{-1}, and RMSE for computation of STT of Gamma and Sigma are, respectively 187 hours and 209 hours. The results of Sigma and Gamma models in convex-convergent hillslope were close.

The results of saturation models of Gamma and Sigma under recharge rates 135 to 155 mm day^{-1} in straight-convergent hillslope were almost the same (Fig. 3d). RMSE values in models Gamma and Sigma for computation of SZL were, respectively 2.8 mm day^{-1} and 3.3 mm day^{-1}, and for computation of STT were 24 mm day^{-1} and 12 mm day^{-1} respectively (Fig. 4d). This shows that the three models are in relative agreement in estimation of SZL and STT of straight-convergent hillslopes. For this hillslope too, the Gamma and Sigma models have good conformity.

Table 2. Parameters of Gamma and W models

Hillslope Number	B	CR	a	c	β	γ
1	1.2	14.39	-0.016	50	-0.39	0.001
2	1.2	1	0	30	-0.39	0.001
3	1.2	0.0695	0.036	3	-0.39	0.001
4	0	7.389	-0.016	50	-0.27	0
5	0	1	0	30	-0.27	0
6	0	0.135	0.036	3	-0.27	0
7	-2.5	14.39	-0.016	50	-0.1	-0.001
8	-2.5	1	0	30	-0.1	-0.001
9	-2.5	0.0695	0.036	3	-0.1	-0.001

Table 3. Values of RMSE in Gamma and Sigma models

Hillslope Number	Profile Curvature	Plan Shape	SZL [m] Gamma	SZL [m] Sigma	STT [hours] Gamma	STT [hours] Sigma
1	Concave	Convergent	1.20	1.1	172	172
2	Concave	Parallel	59.3	52.0	135	104
3	Concave	Divergent	4.60	5.50	15.4	25.5
4	Straight	Convergent	2.80	3.30	24.0	12.0
5	Straight	Parallel	1.40	0.74	0.11	2.63
6	Straight	Divergent	--	6.00	--	6.20
7	Convex	Convergent	32.1	31.3	187	209
8	Convex	Parallel	4.30	9.0	3.60	10.3
9	Convex	Divergent	--	31.4	--	30.0

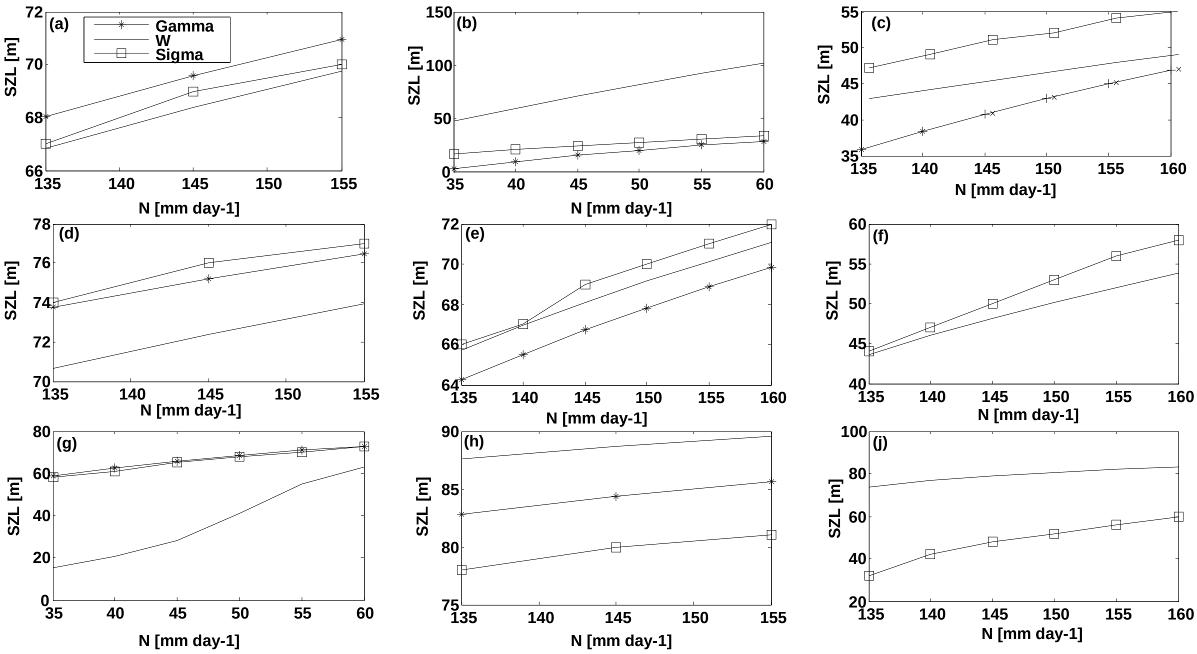

Fig. 3. Saturation zone length [m] in the hillslopes defined in Fig. 2 and Table 2; a) to i) correspond to hillslope numbers 1 to 9.

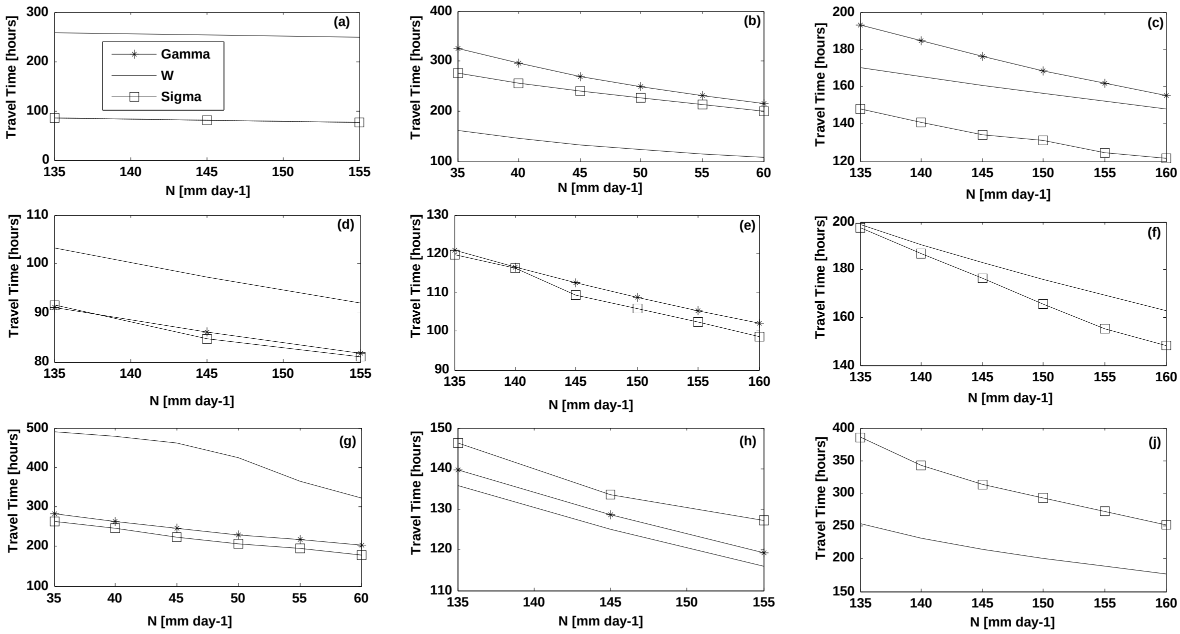

Fig. 4. Subsurface travel time [hours] in the hillslopes.

The results concerning SZL in the three models for concave-convergent hillslopes were similar (Fig. 3a). The values of RMSE in Gamma and Sigma models were, respectively 1.18 m and 1.1 m. As for the travel time, these two models showed total agreement while having many discrepancies with the W model (Fig. 4a). The value of RMSE of both Gamma and Sigma is 172 hours. Generally, for convergent hillslopes with different curvatures the model Gamma showed results very similar to those of the model Sigma.

For concave-parallel hillslopes, RMSE of SZL for the Gamma and Sigma models are, respectively, 59.3 m and 52 m, and the values of RMSE for evaluation of travel time are, respectively, 135 hours and 104 hours. In this type of hillslope the results for the two models are in accord (Figs. 3b, 4b). In convex-parallel hillslopes, the maximum values of SZL corresponded to the W model running in the range of 87 to 90 m, and the minimum values were of the Sigma model between 78 m and 82 m (Fig. 3h). The results of Gamma model are similar to the averages of the other two models. The values of RMSE and of SZL for Gamma and Sigma model were 4.3 m and 9 m, respectively, and RMSE's for evaluation of STT were respectively, 3.6 hours and 10.3 hours (Fig. 4h). For straight-parallel hillslopes, RMSE and SZL values for Gamma and Sigma models are 1.4 m and 0.74 m respectively, and RMSE's for computation of travel time as 0.11 hours and 2.63 hours respectively. According to the results, all three models are in agreement regarding results for straight parallel hillslopes (Figs. 3e, 4e).

Finally for concave divergent hillslopes (Figs. 3c, 4c), the values concerning RMSE of SZL for Gamma and Sigma models are, respectively 4.6 m and 5.5 m, and those of RMSE for computation of travel time are 15.4 hours and 25.5 hours, respectively. By the obtained results it could be deduced that the Gamma model shows a better conformity with the W model for concave divergent hillslopes than the Sigma model.

The results of Gamma model for convex divergent hillslopes did not converge to a solution so the model could not evaluate appropriate results for these hillslopes (Figs. 3j, 4j). Having the RMSE value as 31.4 hours, Sigma model had significant differences in the results compared with W model. The results of Gamma model for straight divergent hillslopes did not converge to a solution (Figs. 3f, 4f).

Conclusions

In this study, a new saturation model, called Gamma model, based on a combined topography model, was introduced to separate the saturated and unsaturated zones and estimate subsurface travel time in complex hillslopes. The Gamma model is capable of considering the effects of plan shape and profile curvature upon saturation rate as well as subsurface travel time. Then, the results of three saturation models, Gamma, Sigma and W were compared. The geometric equations of hillslopes such as plan shape and profile curvature were different in each model. Gamma model has a simpler geometry and its results were compared to those of the other two models.

In Sigma and W models, the SZL was obtained by solving the saturation equations but in Gamma model, a new analytical equation was introduced to calculate the SZL in complex hillslope.

According to the results, the reactions of complex hillslopes in the three models are not the same. In Gamma or Sigma models, in general, in nine complex hillslopes no definite course of reaction was detected. In most hillslopes, the two models Gamma and Sigma had coinciding results and in convex divergent and straight divergent hillslopes, Gamma model was not efficient because the saturation equation of Gamma did not converge to a real solution.

The results of the three models were relatively close to each other in convergent and parallel hillslopes of different profile curvature type. However, due to the existence of an analytical equation for estimation of saturated zone length in the Gamma model, this model is recommended. It should be noted that for straight divergent and convex divergent, Gamma model is not suitable and Sigma or W model should be used. Undoubtedly, the most efficient model will be identified only by conducting experimental studies using laboratory models.

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