

**Analysis of long-term trends and probability characteristics of low-flow
in Low Tatra Mountains**

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Analyzing variations in streamflow over a long period is crucial from the perspective of water resources management and understanding the impacts of climate change on the basin. Changes in the frequency, magnitude, and seasonality of hydrological extremes are anticipated consequences of climate change. Statistical trend analysis identifies positive or negative variations within a dataset, enabling the projection and prediction of future projections. The objective of this study is to examine temporal changes in the magnitude of minimum discharge series at four selected pristine mountain catchments within the National Park of Low Tatras (NAPANT) region. Four series of long-term daily discharge data spanning from 1930/31 to 2019/20 were statistically analyzed: Vajskovský brook: Dolná Lehota, Bystrianka: Bystrá, Štiavnička: Mýto pod Ďumbierom, and Boca: Kráľova Lehota. T -year minimum specific runoff q_{min} was processed using files containing annual minimum flows from 1- and 7-day minima per hydrological year. For the analysis of long-term precipitation trends, datasets of annual precipitation totals from stations Banská Štiavnica (575 m a.s.l.), Brezno (487 m a.s.l.), and Vyšná Boca (930 m a.s.l.) were utilized. Across all catchments, a similar long-term trend in minimum specific discharges was observed: a decline during the period 1931–1965, decadal minima around 1983–1992, and a slight increase post-1991 (with a significant increase in Boca). This trend correlates with the long-term precipitation trend. The lowest decadal precipitation totals at the respective stations occurred during the years 1982–1991 ± 1 year.

KEY WORDS: small high mountain basins, T -year minimum discharge, long-term trends

Introduction

The question of potential changes in the hydrological regime of rivers is a topic that is frequently discussed, particularly in connection with the anticipated climate change or more recently promoted theories regarding land drying. The determination and assessment of T -year minimum discharges and low water content at water measuring stations in Slovakia were governed by OTN 3113-1 (2007), with the current proposed regulation being OTN ŽP 3113-1:04. Older studies pertaining to the characterization of small water bodies and low water content in Slovak streams can be found in the works of Szolgay (1977), Drako and Majerčáková (1989), Balco (1990), Majerčáková et al. (1995; 1997), Lešková (1997), Grešková (1998; 1999), or Demeterová and Škoda (2004; 2009), as well as more recent works by Fendeková and Fendek (1999), Fendeková et al. (2014; 2017) or Sabová and Kohnová (2023). In the monograph, Lešková (1997) published probability characteristics of low flow in 20 water gauging stations in Slovakia, which were selected for the FRIEND project. Utilizing approximately 60 years of data series for 1-, 7-, and 15-day minimum series, probability values were estimated.

The objective of the work by Blaškovičová et al. (2017) was to highlight the potential use of quantiles of M -daily discharges from small water body areas as inputs for establishing ecological discharges. Also taking into account the associated impacts such as the duration of periods during which natural decreases in discharge below the threshold value could lead to restrictions on water usage.

In the Czech Republic, even regions with generally higher annual precipitation amounts, such as the mountains along the Czech borders, experience drier periods from a long-term perspective based on instrumental records (Brázdil et al., 2015; Fiala et al., 2010). According to Vlach et al. (2021), recent years have suggested an increasing risk of drought occurrence and restricted water supply, even in usually humid mountain regions. Their research focused on evaluating regional climate change and its effects on hydrological drought characteristics in headwater areas in the Ore Mountains along the Czech/German border. The primary aim was to assess and compare changes in streamflow regimes and trends in long-term hydrological and climatic time series within selected sub-catchments. Analyzing the series of average annual flows for

the period 1900–2000, Pekárová (2003) discovered that the flow series of Slovak streams contain a multi-year cyclic component. The findings indicate that when determining long-term flow trends, it is essential to consider approximately 13–15-year cycles of alternating dry and wet periods, and trends must be assessed for periods of complete cycles (from minimum to minimum or from maximum to maximum). In the monograph Stănescu (2004), regional analysis of the annual peak discharges in the Danube River catchment was presented. Trends in average, maximum, and minimum annual flows for the period 1877–2013 on the Danube in Bratislava were analyzed by Blaškovičová et al. (2013). They observed a balanced trend in average annual flows, while annual minimums exhibited a decreasing trend. In the same water measuring station for the period 1877–2005, Pekárová et al. (2002) studied trends of maximum 30-day discharges (characteristic of high water content) and 330-day discharges (characteristic of low water content). The results revealed a slight increase in minimums and a slight decrease in maximums.

Poárová et al. (2013) examined trends in minimum annual and monthly flows for the period 1961–2012 in Slovak basins. According to their findings, annual minimums are decreasing in the basins of the Morava, Lower Váh, Nitra, Hron, Ipel', Slaná, and Bodva. Conversely, streams in the Upper Váh, Poprad, Hornád, and Bodrog basins exhibit an increasing trend in annual minimums. Pekárová et al. (2017) compared trends in annual minimum and maximum flows on ten significant Slovak streams across two periods (1931–1972 and 1973–2014). Additionally, authors Šimor and Lupták (2021) addressed the development of trends in minimum annual discharges and average annual discharges for the period 1961–2000 (utilized by the Slovak Hydrometeorological Institute (SHMI) as a reference period since 2006) and compared them with trends for the period 1961–2015. Upon evaluating both periods, they conducted a mutual comparison and subsequent analysis of potential changes. According to their results for the minimum annual discharges at $p=0.05$ significance level the detected trend for the period 1961–2015 was rising at Boca station (not significant for the period 1961–2000). For the Vajskovský brook, Bystrianka and Štiavnička no significant trend was detected in both periods. The authors Ďurigová et al. (2019) and Blaškovičová et al. (2022) also dealt with the analysis of changes in average monthly flows in Slovakia in recent decades. The time data series were subjected to basic descriptive statistic, trend and periodicity analyses, and AR-ARCH model. Methods were applied to five larger basins in Slovakia and seven smaller basins located in the northern part of the Váh River basin. The results show greatest changes (in trend and periodic analysis) in the Hron River. Mountainous environments are particularly vulnerable to climatic warming, and long term observations suggest a shift of snow-influenced river discharge towards earlier periods of the year. For water resources management, the seasonal patterns of discharge are particularly relevant, as the shift to lower flows in summer and autumn combined with increased water demand could

lead to water shortage in downstream catchments. In alpine regions, monthly runoff and the associated occurrence of flow extremes are characterized by a strong seasonality, with maximum runoff typically occurring in spring and summer during the snowmelt season and minimum runoff in winter. Changes in flow magnitudes and seasonality in alpine environments can have wide-reaching socio-economic and ecological implications. The results of the scientific works predict that annual low flows will increase in the Alps, as winter low flows will increase due to changes in snow dynamics related to increased temperatures. (Arnoux et al., 2021; Hanus et al., 2021, Parajka et al., 2016, Muelchi et al., 2021). In Austria, studies project an increase in winter flows and a decrease in summer flows in the 21st century, with the largest increases in winter flows found in high-elevation areas.

In this study, we focused on long-term trends and variability in the magnitude and timing of minimum discharge series at four relatively unaffected mountain catchments in the upper Hron and Váh river basins (Vajskovský brook: Dolná Lehota, Bystrianka: Bystrá, Štiavnička: Mýto pod Ďumbierom, and Boca: Kráľova Lehota) with 90-year time series spanning from 1930/31 to 2019/2020. The aim was to formulate a methodology for processing T -year 7-day minimum discharges using a single type of theoretical distribution function with the potential for regionalization (generalization) of its parameters. Such a method could potentially serve as a foundational framework for a standardized approach to evaluating minimum flows and fundamental characteristics of small water bodies, essential for the planning, construction, and operation of water management facilities and structures along streams for effective water resource management. The utilization of a single type of distribution function also allows for the estimation of distribution function parameters and, consequently, the determination of T -year minimum discharges for stream sections lacking direct observations.

Material and methods

Methods

The analysis of hydrological series necessitates that the provided series be homogeneous and stationary. Therefore, when selecting stations, it is crucial to choose those where no alterations have occurred that would impact the water quantity and the hydrological flow regime.

Data can undergo statistical analysis at various intervals (hourly, daily, monthly, seasonal, annual, irregular). The time series of the hydrological element serves as the fundamental basis for evaluating the hydrological process regime. From a statistical perspective, the measurement outcomes are regarded as a random sample comprising n elements, which are utilized to ascertain the properties and characteristics of the investigated random variable.

To assess low flow periods, following characteristics are usually evaluated:

- **flow characteristics** – minimum average daily flow rate (monthly and annual step, for the entire period), M -daily flow (curve of exceeding average daily flow), minimum monthly and annual flow rates, T -year minimum discharges;
- **failure characteristics** – time occurrence of dry periods (occurrence date, number of days of low flow period, longest dry season) and insufficient volumes.

In this study, we concentrated on evaluating both types of characteristics. The individual flow characteristics were computed from the series of average daily discharges. Annual minimum discharges from 7-day minimum per winter-spring (November–April) and summer-autumn (May–October) season of the hydrological year were used for estimating seven day T -year minimum discharges $Q_{7d,min}$ and specific runoffs $q_{7d,min}$. The 7-day minimums are taken from moving averages of the daily discharges calculated for every possible season that is completely within the hydrological year.

For the estimation of T -year series of 7-day minimum specific discharge $q_{7d,min}$, we used a theoretical Log-Pearson Type III probability distribution. (LP3). By adjusting the skewness coefficient of this distribution, it is possible to enhance the estimation in the lower range of values. Due to the potential for regionalization of distribution parameters, we recommend selecting one distribution and processing all data while regionalizing the parameters accordingly.

The LP3 distribution is a three-parameter gamma distribution with a logarithmic transformation of the variable. It is widely employed for flood analysis because the data frequently align with predicted annual extreme flow series. The LP3 probability distribution is utilized for estimating extremes in numerous natural processes and is the most commonly utilized probability distribution, particularly in hydrology (Griffis and Stedinger, 2007a; b; Koutsoyiannis, 2005). Pilon and Adamowski (1993) devised the Logarithmic Likelihood Function LP3, which includes parameter estimation. In Cheng et al. (2007), a method based on the frequency factor for hydrological frequency analysis is outlined for the random generation of five distributions (normal, lognormal, extreme value Type I, Pearson Type III, and Log-Pearson Type III).

The method described in Bulletin 17-B (IACWD, 1982) was used to estimate the distribution parameters. The LP3 division has been used. The probability density function of the type III Pearson distribution has the following form:

$$f(X|\tau, \alpha, \beta) = \frac{\left(\frac{X-\tau}{\beta}\right)^{\alpha-1} \exp\left(-\frac{X-\tau}{\beta}\right)}{|\beta| \Gamma(\alpha)} \quad (1)$$

$$\frac{X-\tau}{\beta} \geq 0,$$

where:

τ – location parameter;

α – shape parameter;

β – scale parameter;

$\Gamma(\alpha)$ is the Gamma function given by:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} \exp(-t) dt. \quad (2)$$

The Kolmogorov-Smirnov test was performed to test the assumption that the discharge magnitudes follow the theoretical distributions. The p-value ($\alpha \geq 0.05$) was used as a criterion for the rejection of the proposed distribution hypothesis. The second criterion was the comparison between the theoretical distribution and the empirical distribution of the data series. In this work we used Cunnane formula.

Study area and data

In this study, we focused on the analysis of the development of minimum flows of alpine streams with 90-year series of observations from the Low Tatras National Park (NAPANT). Even though it was only declared in 1978 as the third national park in Slovakia, there were already several protected areas in this territory a decade earlier. Therefore, we can consider these flow ranges to be relatively unaffected. In the Low Tatras, there are several primeval forests that are not open to the public.

A series of 90-year series (1931–2020) of average daily discharges from three mountain subbasins of the Hron River were used as input data: 1. Vajskovský brook (Dolná Lehota), 2. Bystrianka (Bystrá), 3. Štiavnička (Mýto pod Ďumbierom) from southern slopes of the Low Tatras. A data of daily discharges from the Boca stream from the Kráľová Lehota station (4.), which flows into the Váh River, were used from the northern slope. The location of the selected sub-basins in the territory of Slovakia is illustrated in Fig. 1.

The highest meteorological station in NAPANT is Chopok at an altitude of 2005 m a.s.l. in the Bystrianka basin, with an average annual air temperature of -1.2°C . The basin parts (valleys) of the National Park are characterized by a moderately dry to humid climate, with a large temperature inversion, January temperatures drop from -3.5° to -6°C , July rises from 14.5° to 18°C . Thanks to frequent temperature inversions (warm sunny weather prevails in the mountains, while low temperatures remain in the basins under a layer of fog or low cloud cover), the highest temperature amplitude is precisely in the basins, where an extremely low value of -38°C was measured in Liptovský Hrádok, and the absolute maximum in Brezno reached 36.5°C .

The amount of atmospheric precipitation, like the air temperature, is generally affected by the altitude. While the annual total exceeds 1,400 mm in ridge positions (up to 1,600 mm in Chopok), it is less than 900 mm in the valleys.

The catchment of Bystrianka brook, which was in 1962 selected as representative for the seasonal snow cover evolution in the higher mountains of central Slovakia, is the most studied (Babiaková and Bodiš, 1987; Holko, 2000; and Pecušová and Holko, 2002). The Bystrianka basin belongs to the Low Tatras crystalline. The catchment area up to Bystrá gauge is 36.1 km^2 .

The height range is relatively large (580–2043 m a.s.l. – Ďumbier peak). The area is dominantly built by gneiss. Except a ski resort, there was no permanent settlement in catchment. The catchment is covered by the mixed beech and spruce forest at lower altitudes and by spruce at higher altitudes, (spruce represented 60% of the forest). The altitude of the forest line is 1500 m a.s.l. Dwarf pine and Alpine meadows occur above the forest line.

The Vajskovský brook originates on the southern slopes of the Low Tatras and flows through the Vajskovská valley to the south to Hron and the basin has an area of 59 km². It originates in the Low Tatras at an altitude of approx. 1,680 m a.s.l. It flows into the Hron in the territory of the village of Dolná Lehota, in the inner village of its local part of Lopej, at an altitude of approx. 438 m a.s.l. The Vajskovský stream belongs to the Central Highlands region with a snow-rain type of runoff. It has the highest water level during the period of snow melting in the mountainous areas of the Low Tatras, i.e. in April, May and June, while the maximum flow usually occurs in May. The lowest discharges in the Vajskovský brook were always reached in the coldest months of January and February. The hydrological conditions of the Vajskovský stream in the treated area are significantly influenced by the upper part of the basin located in the Low Tatras, where the annual precipitation is 1000–1600 mm and the snow cover here lasts for more than 180 days. The differences between maxima and minima are much smaller than in other, especially flysch

basins of Slovakia.

The Štiavnička stream originates in the Low Tatras below the Čertovica saddle (1,238 m a.s.l.) at an altitude of around 1,180 m a.s.l., the stream flows mainly to the southwest. It flows into the Bystrianka in the territory of the village of Bystrá at an altitude of 558.8 m a.s.l. and the catchment area is 54 km². Štiavnička is a Low Tatras stream in the Brezno district, it is a left-hand tributary of the Bystrianka and has a length of 13.5 km. Štiavnička is typically a mountain stream with a torrential character, on the upper stream it forms a uniform valley with springs of mineral springs. According to SHMI (2010), at the tributaries of the Hron in the assessed stations, the smallest average discharges occurred during the summer-autumn depression, or in winter. The number of occurrences of the smallest average daily discharges in individual months is greatest in the period 1981–1990, except for Štiavnička, where the occurrence is spread over the entire period of observation.

Boca (sometimes Bocianka) is a river in Liptov, in the territory of Liptovský Mikuláš district. It is a left-hand tributary of the Váh River with a length of 18.5 km. It has a snow-rain drainage system. It originates in the Low Tatras, in the geomorphological subdivision of the Ďumbier Tatras, at an altitude of approximately 1,400 m a.s.l. It flows into the Váh in the territory of the village of Kráľova Lehota at an altitude of approx. 658 m a.s.l. The basic characteristics (SHMI, 1963) of the given basins are listed in Table 1 a, b.

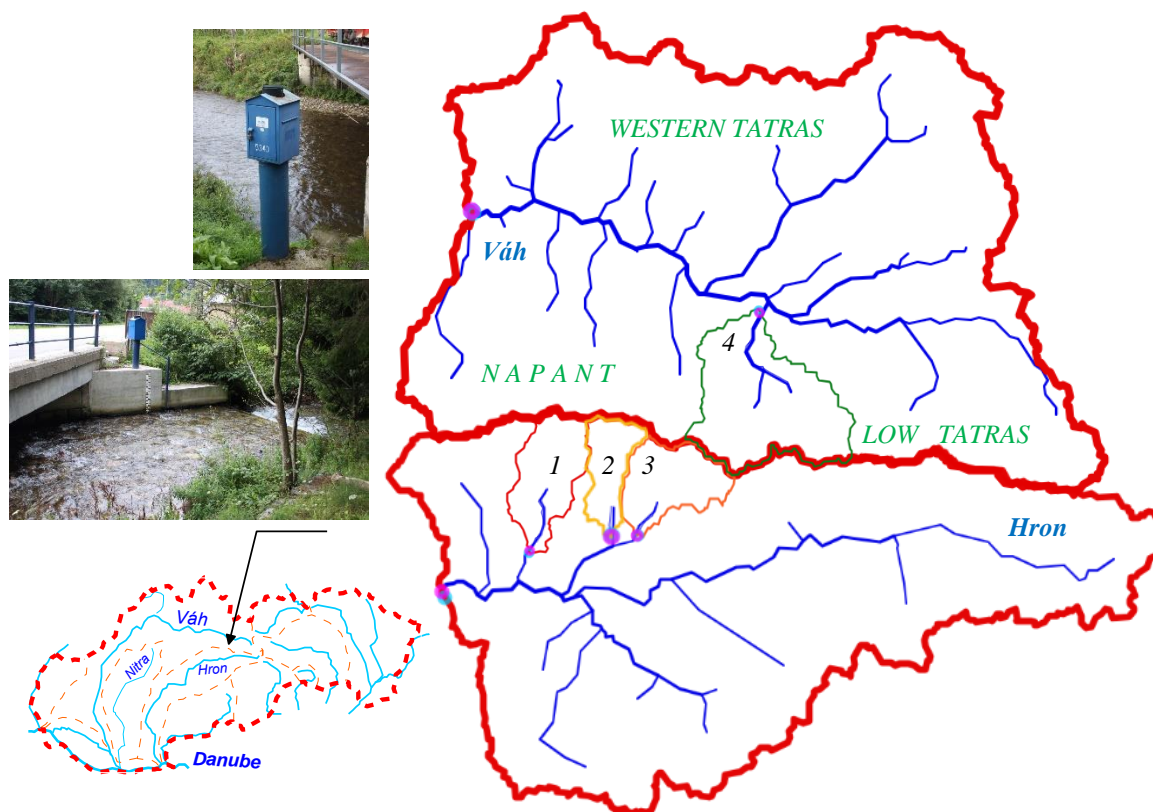


Fig. 1. The study area: Upper Váh and Hron River basins in Low Tatras mountains, Slovakia. Photo: water gauge Boca: K. Lehota and Bystrianka: Bystrá (Photo: Meszároš, 2019).

Table 1a. Basic characteristics of selected basins with long series of average daily discharges in the upper Hron and Váh River basins

No.	DB No	Hydrological station	Stream	Basin area A [km ²]	Length of the basin L [km]	Forestry [%]	A/L^2	Gauge altitude [m a.s.l.]
1.	7070	Dolná Lehota	Vajskovský	53.02	15	70	0.24	495.27
2.	7060	Bystrá	Bystrianka	36.01	12	80	0.25	574.43
3.	7065	Mýto pod Ďumbierom	Štiavnička	47.1	10.9	80	0.4	616.75
4.	5340	Kráľova Lehota	Boca	116.6	18.3	80	0.35	655.08

Table 1b. Basic hydrological characteristics of selected catchments for the period 1931–1960 are represented as follows (Characteristic hydrological data of Slovak streams, 1963): P represents precipitation, R represents runoff depth, ET denotes balance evapotranspiration, k signifies the runoff coefficient, q stands for specific runoff, and Q represents discharge

Stream	No.	P [mm]	ET [mm]	R [mm]	k -	q [l s ⁻¹ km ⁻²]	Q [m ³ s ⁻¹]
Vajskovský	1.	1206	397	809	0.65	25.65	1.36
Bystrá	2.	1202	422	780	0.65	24.72	0.89
Štiavnička	3.	1180	389	791	0.67	25.05	1.10
Boca	4.	1073	464	609	0.57	19.3	2.25

When analyzing the long-term precipitation trends, we utilized annual precipitation totals from the Banská Štiavnica (575 m a.s.l.), Brezno (487 m a.s.l.), Chopok (2005 m a.s.l.), and Vyšná Boca (930 m a.s.l.) stations. Data from the period 1901–1970 were sourced from the study conducted by Šamaj and Valovič (1978).

Results and discussion

Analysis of the average daily flow regime

Firstly, we focused on analyzing the hydrological regime of the average daily discharges from the selected four water gauging stations (Fig. 2a). From the plotted values of daily data, it can be observed that non-homogeneous values of daily discharges for the period 1931–1941 are apparent in the Bystrianka brook. The daily values of the flows at this station were therefore before further calculations homogenized for the given period using hydrological analogy based on the neighbouring streams. For the visual inspection of minimum flows (below Q_{355} – probability of exceedance of 97.2% calculated for period 1931–2020), we utilized graphs with a logarithmic scale on the y-axis (Fig. 2b). On this graph, the occurrence of extremely dry periods is clearly visible. Basic hydrological characteristics of the average daily discharges for the entire 90-year period from 1931 to 2020 were calculated at the selected stations. The results

are presented in Table 2. A decreasing trend of average daily discharges, as well as multiannual variability during the period 1931–2020, were observed at all stations.

The long-term average specific runoff in the Bystrianka stream basin at the Bystrá station for the entire period is the highest – 26.50 l s⁻¹ km⁻² – which is a logical result considering that this basin is the smallest.

The annual patterns of average daily discharges – percentiles $P10$ (left), $P50$, and $P90$ (right) – are depicted in Fig. 3. A comparison of the tenth percentiles for three periods (1931–1960, 1961–1990, and 1961–2020) is illustrated in the left portion of Fig. 3. These graphs enable a visual assessment of changes in the annual progression of minimum discharges for each day throughout the year.

In the first three basins (Hron River basin) in the period 1931–1960, $P10$ flows are significantly higher. There can be several reasons:

1. Higher precipitation totals in basins, lower evapotranspiration due to lower air temperatures, changes in vegetation;
2. Discrepancies in the measurement of water conditions (freezing in the water measuring station during winter, obstacles in the riverbed);
3. Extrapolation of the rating curve to the lowest water level values, changes and relocation of the flow profile;
4. Higher withdrawals of water from streams.

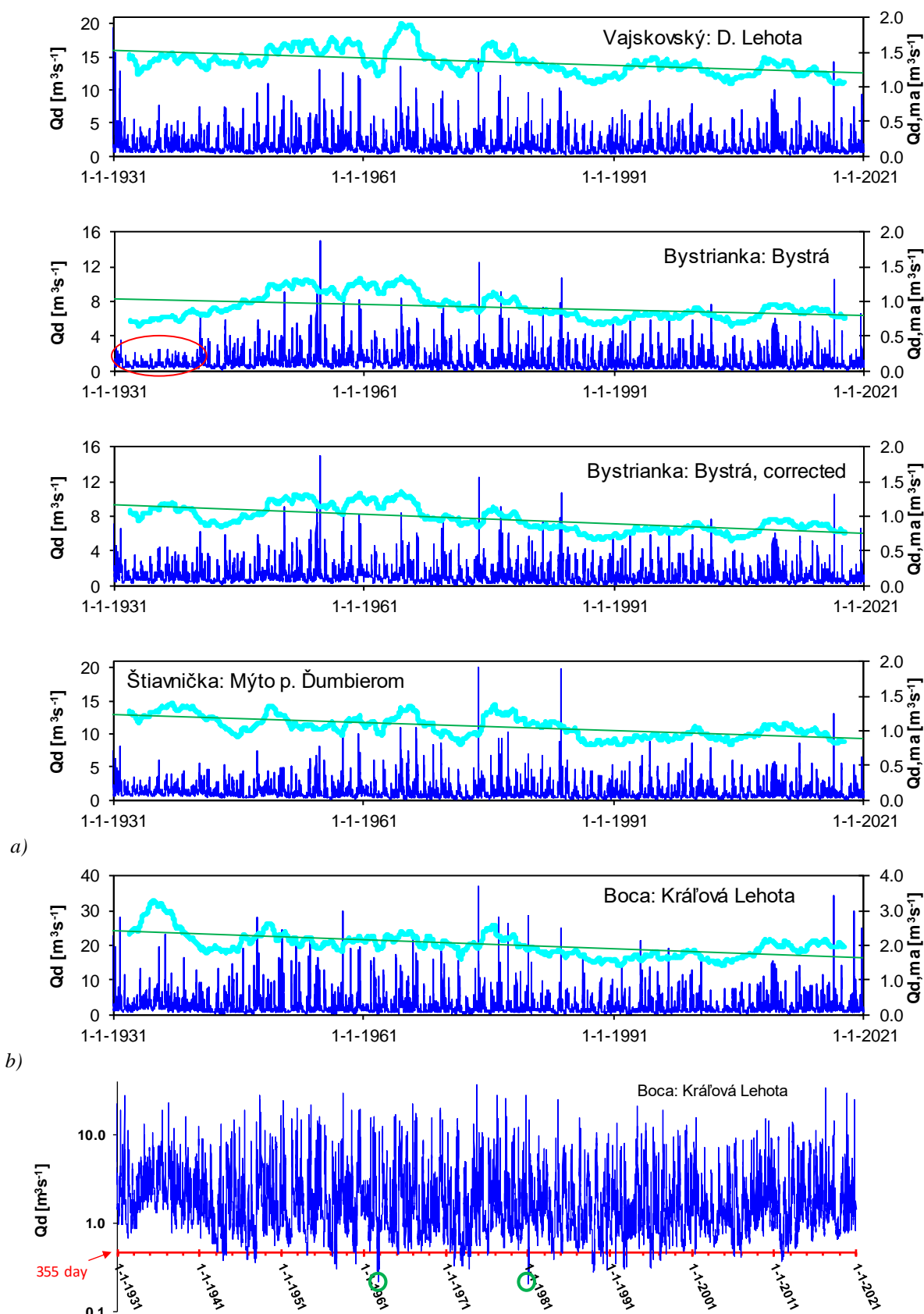


Fig. 2. a) Course of the average daily discharges (dark blue lines) 4-years moving averages of the daily discharges (light blue lines) with long term linear trend of the selected stations, 1931–2020 period. b) Course of the average daily discharges (blue lines), values below 355-day discharge, Boca brook, 1931–2020 period. Extreme low discharge – green circle.

Table 2. Basic hydrological characteristics of the daily discharge series, whole period 1931–2020

No	Stream	Q_{mean}	q	R	c_s	c_v	$Q_{min,d}$	$Q_{max,d}$	$P90.4\%$	$P97.2\%$
		$[m^3 s^{-1}]$	$[l s^{-1} km^{-2}]$	$[mm]$			$[m^3 s^{-1}]$	$[m^3 s^{-1}]$	Q_{330d} $[m^3 s^{-1}]$	Q_{355d} $[m^3 s^{-1}]$
1.	Vajskovský	1.36	25.57	806	3.04	0.88	0.25	20	0.47	0.37
2.	Bystrianka	0.96	26.50	838	2.82	0.88	0.10	15	0.29	0.20
3.	Štiavnička	1.05	22.35	704	3.36	0.93	0.17	20	0.32	0.25
4.	Boca	2.01	17.26	544	4.05	1.03	0.21	37	0.62	0.46

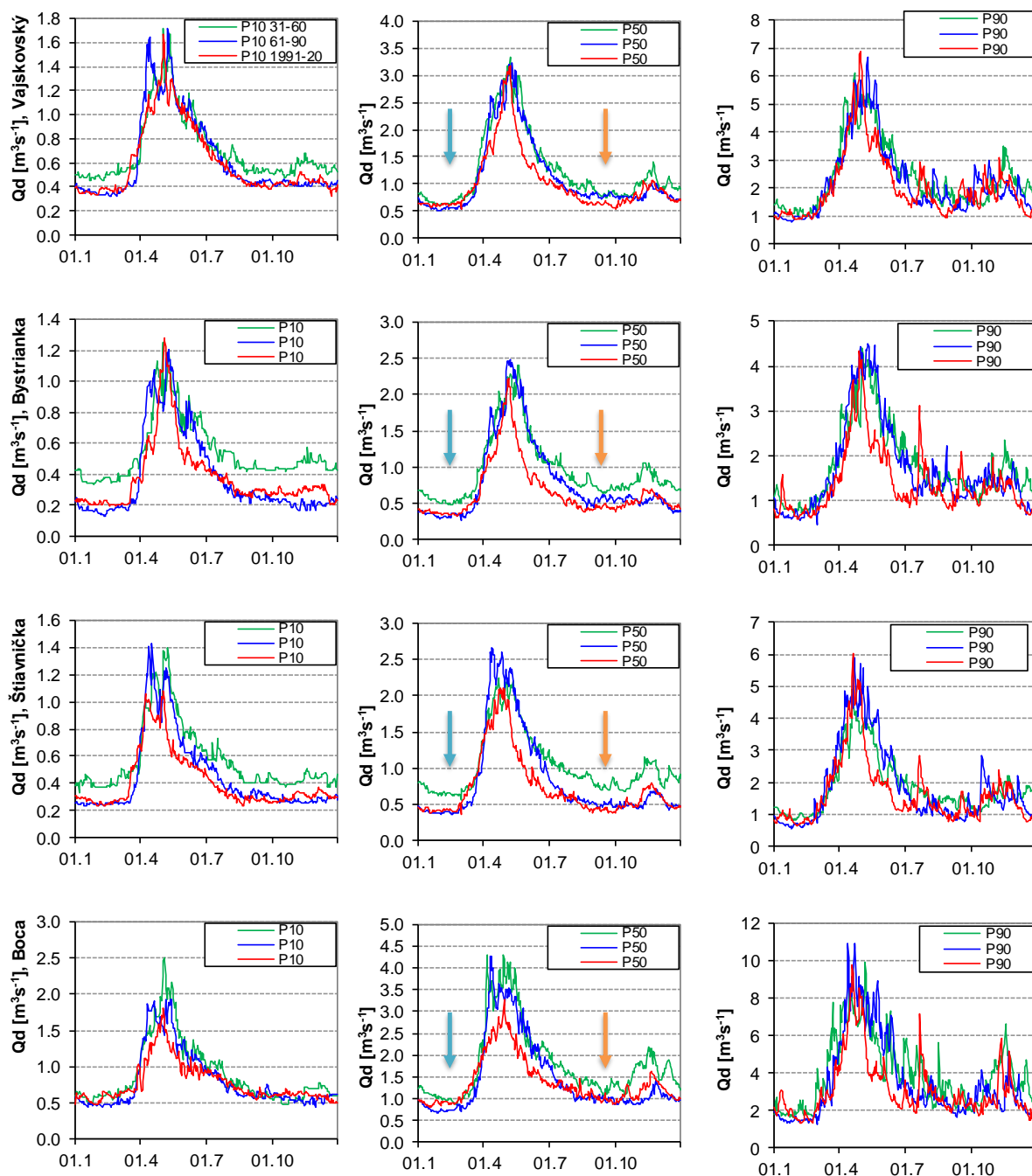


Fig. 3. Annual course of discharge percentiles P10 (left), P50 and P90 (right). Comparison of the periods 1931–1960 (green line), 1961–1990 (blue line), and 1991–2020 (red line). Arrows show the occurrence of minimum flows: winter minimum (blue arrow); summer minimum (orange arrow).

Drako and Majerčáková (1989) found a significant dependence of minimum flows on precipitation for the previous 6 months. Therefore, we proceeded to compare the trends and multi-year variability of annual series of minimum specific runoff with precipitation.

Analysis of the long-term trends and variability of the minimum annual specific runoff and precipitation

From the series of average daily flows, we constructed the series of minimum specific runoff for calendar years. The series of minimum specific outflows from Vajskovský brook is depicted in Fig. 4. During the years 1931–1960, the decadal averages reached $9.6 \text{ l s}^{-1} \text{ km}^{-2}$, in the years 1961–1990 the minimum specific outflows decreased significantly, the driest decade was 1983–1992 ($5.94 \text{ l s}^{-1} \text{ km}^{-2}$). In the last thirty years, minimum specific outflows have grown slightly. The development of annual precipitation totals is presented in Fig. 5.

The autocorrelation analysis of the annual series of precipitation and minimum flows, an approximately 3.6-, 14- and 29-year cycle of value fluctuations were detected. In all investigated basins, the long-term trend of minimum specific runoff was similar, a decrease in the period 1931–1965, a decadal minimum around the years 1983–1992, a slight increase after 1991 (the increase is significant in Boca). This trend is related to the long-term precipitation trend (Drako and Majerčáková, 1989; Balážovičová et al., 2023; Pavelková et al., 2023). When analysing the long-term development of precipitation, we used the series of annual precipitation totals from the stations Banská Štiavnica (575 m a.s.l.), Brezno (487 m a.s.l.) and Vyšná Boca (930 m a.s.l.). Unfortunately, the Chopok station was put into operation only in 1955. The lowest decadal precipitation totals in the given stations occurred in the years 1981–1990 (± 1 year).

T-year minimum specific runoff series for two seasons

According to the fact, that our future work will focus on analyzing minimum flows in relation to water temperature, we calculated 7-day minimum discharges

$Q_{7d,min}$ and specific runoffs $q_{7d,min}$ for the winter-spring (WS) season (November–April) and summer-autumn (SA) season (May–October) of the hydrological year spanning from 1930/31 to 2019/20. Fig. 6 presents selected example time series for Štiavnička brook and also days of occurrence of minimum annual discharges in a given hydrological year. At the Štiavnička station, both 1-day and 7-day minimum flows exhibit a decreasing trend until 1965, followed by an increasing trend since 1990, for both the winter WS and summer SA seasons. The basic statistical characteristics of these series are outlined in Table 3.

In the final section, we focused on constructing theoretical probability curves for 7-day minimum specific runoff ($q_{7d,min}$) for selected stations for both the winter-spring and summer-autumn periods. The example of the theoretical LP3 curves of non-exceedance for the series of 7-day minimum specific runoff ($q_{7d,min}$) for Vajskovský brook is depicted in Fig. 7a. We constructed the curves with attention to the lower values so that the empirical and theoretical values in the lower part of the graph align as closely as possible. This was achieved by adjusting the alpha parameter in equations (1, 2). In the case of the Boca stream, we identified one low outlier (Fig. 7b). This value was excluded from the series, and 100-, 50-, and 20-year 7-day minimum specific runoff $q_{7d,minT}$ were estimated from an 89-element series.

The estimated values of T-year 7-day minimum specific runoff are summarized in Table 4.

Theoretical LP3 non-exceedance lines, when used automatically, in some cases significantly underestimate or overestimate T-year minimum flow values. Therefore, in hydrological practice, the theoretical distribution line is usually replaced by an empirical regression function interpolated by points on a logarithmic-probability scale. The calculated flows from the function created in this way better copy the values of the measured flows, but it is clear that the statistical properties are not respected and it is not possible to use them to extrapolate lower probabilities. Therefore, it is necessary to verify the outliers, whether these values are not erroneous, and in that case exclude them from the set of minimum flows.

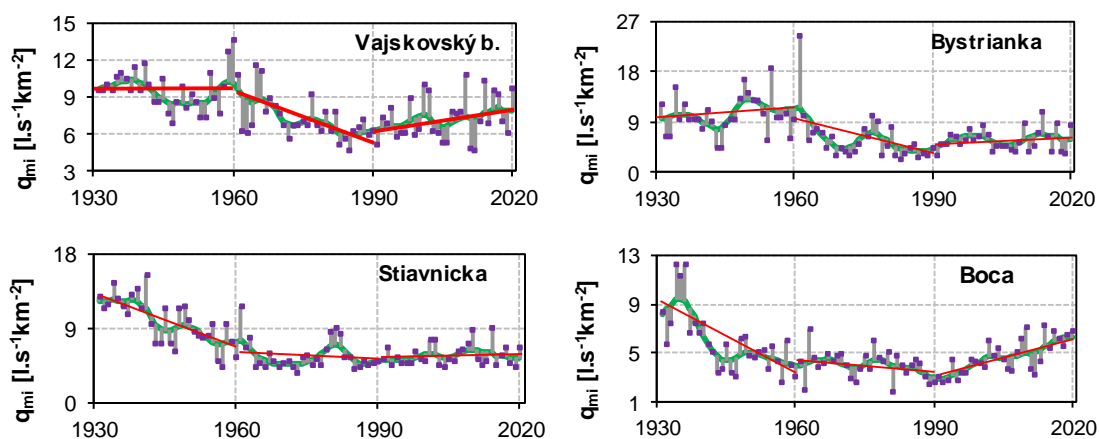


Fig. 4. Long-term linear trends of three 30-year periods (red lines) of the annual minimum specific runoff q_{mi} (lila points), double 7-year moving averages (green curve).

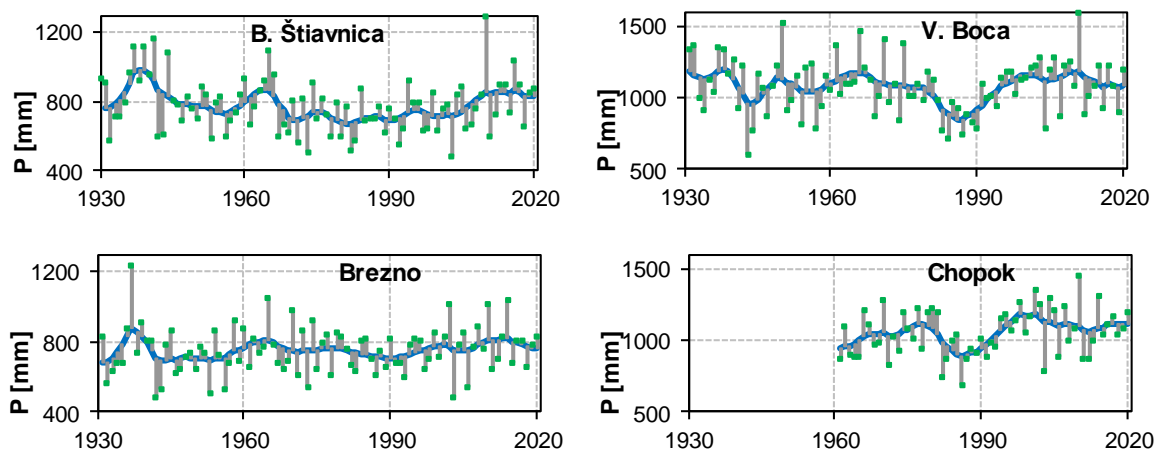


Fig. 5. Course of the annual precipitation totals (green points), 7-year moving averages (blue curve).

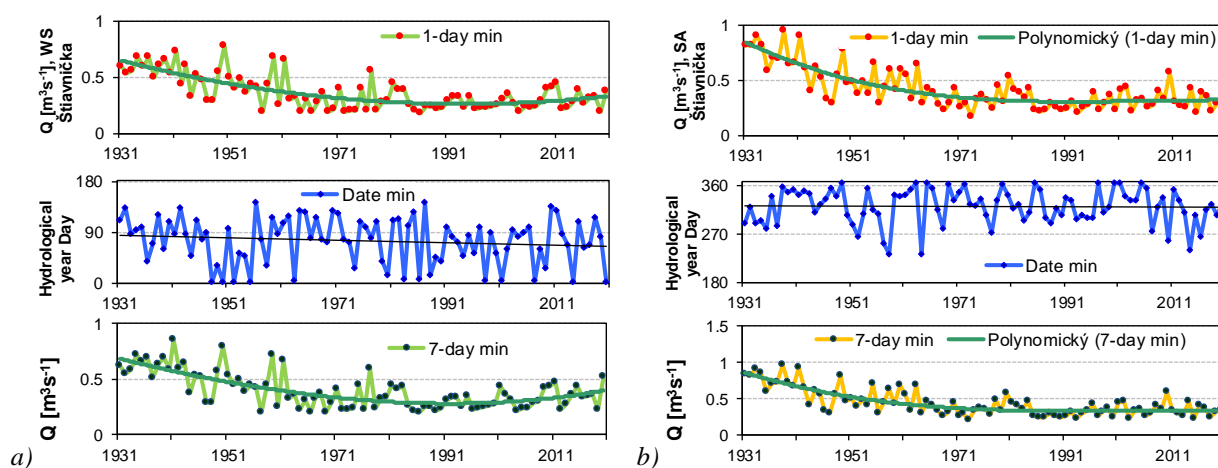


Fig. 6. The course of 1-, and 7-day minimum discharges for the period 1930/31–2019/20 (6a: WS – winter-spring season, 6b: SA – summer-autumn season). Blue graph – days of occurrence of minimum flows in a given hydrological year with linear trend.

Table 3. Basic statistical characteristics of the minimum 7-day series $Q_{7d,min}$, for WS and SA seasons, period 1930/31–2019/20

	VAJSKOVSKÝ		BYSTRIANKA		ŠTIAVNÍČKA		BOCA	
	WS	SA	WS	SA	WS	SA	WS	SA
$Q_{7d,min}$ AVER	0.49	0.57	0.31	0.43	0.39	0.43	0.70	0.74
$Q_{7d,min}$ MIN	0.25	0.32	0.11	0.13	0.20	0.21	0.25	0.23
$Q_{7d,min}$ MAX	1.02	1.18	0.65	1.21	0.85	0.96	1.77	1.67
$Q_{7d,min}$ STDEV	0.13	0.18	0.14	0.21	0.16	0.19	0.31	0.26
$Q_{7d,min}$ VAR	0.02	0.03	0.02	0.04	0.03	0.04	0.10	0.07
Average Date min	21. JAN.	14. SEPT.	20. JAN.	3. SEPT.	13. JAN.	16. SEPT.	19. JAN.	3. SEPT.
Trend Date Slope	–0.23	0.05	0.16	0.35	–0.20	–0.04	–0.12	0.09

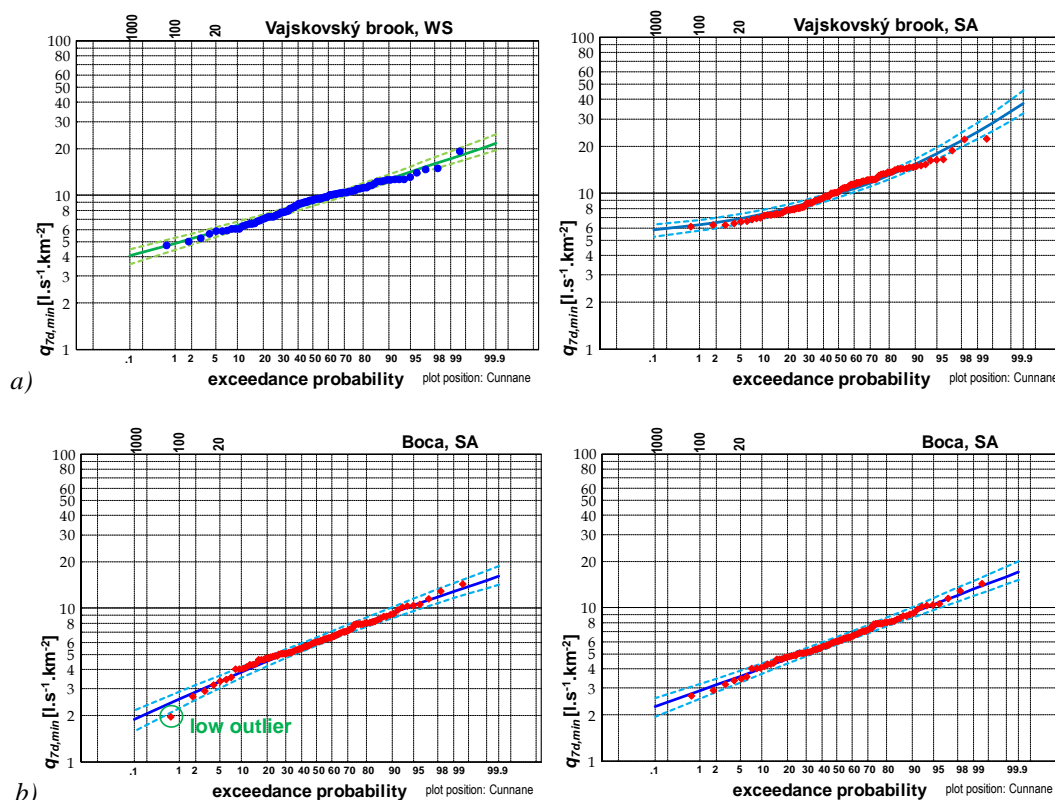


Fig. 8. a) Theoretical LP3 non-exceedance lines of 7-day minimum specific runoff $q_{7d,min}$ for WS (left) and SA (right) periods; 95 and 5% confidence limits, and empirical values (points), Vajskovský brook: Dolná Lehota, period 1931–2020. b) Boca brook, low outlier (left), LP3 curve without low outlier (right).

Table 4. 100-, 50-, and 20-year 7-day minimum specific runoff ($q_{7d,minT}$) in $[l s^{-1} km^{-2}]$ for WS and SA seasons period 1930/31–2019/20. G_w – weighted skew, and s – standard deviation are estimated parameters of the LP3 distribution

T-year	p [-]	WS	confidence	limits	SA	confidence	limits
		$q_{7d,minT}$	$q_{7d,minT}$ (5)	$q_{7d,minT}$ (95)	$q_{7d,minT}$	$q_{7d,minT}$ (5)	$q_{7d,minT}$ (95)
			$[l s^{-1} km^{-2}]$			$[l s^{-1} km^{-2}]$	
		Vajskovský:	$s=0.12$	$G_w=0.12$		$s=0.13$	$G_w=0.87$
100	0.01	4.89	5.29	4.42	6.28	6.76	5.74
50	0.02	5.23	5.63	4.77	6.49	6.97	5.96
20	0.05	5.80	6.19	5.35	6.90	7.37	6.37
		Bystrianka:	$s=0.20$	$G_w=0.60$		$s=0.21$	$G_w=0.20$
100	0.01	3.26	3.69	2.81	3.69	4.25	3.10
50	0.02	3.51	3.95	3.05	4.13	4.71	3.52
20	0.05	3.97	4.42	3.49	4.92	5.53	4.28
		Štiavnička:	$s=0.11$	$G_w=1.14$		$s=0.17$	$G_w=0.98$
100	0.01	4.21	4.62	3.77	4.49	4.95	4.00
50	0.02	4.35	4.75	3.90	4.67	5.13	4.18
20	0.05	4.61	5.02	4.16	5.02	5.48	4.52
		Boca:	$s=0.18$	$G_w=0.19$		$s=0.14$	$G_w=0.08$
100	0.01	2.19	2.48	1.88	2.91	3.20	2.57
50	0.02	2.42	2.71	2.10	3.16	3.46	2.83
20	0.05	2.82	3.12	2.49	3.59	3.89	3.26

Conclusion

In the presented study, the first part focused on analyzing the homogeneity of 90-year records of average daily discharges in small mountainous catchments in the Low Tatras region (1931–2020). Daily discharge values at the Bystrá station: Bystrianka during the period 1931–1941 exhibited non-homogeneity. Therefore, this data were homogenized using hydrological analogy based on neighbouring brooks. We utilized this homogenized dataset for further computations.

In the second part, we concentrated on long-term trend changes and statistical analysis of minimum specific runoff series for the calendar year. During the years 1931–1960, decadal averages of minimum specific runoff decreased, with the driest decade being 1983–1992. In the last thirty years, minimum specific runoff rates have slightly increased. Autocorrelation analysis of minimum flow series revealed cycles of fluctuation approximately every 3.6, 14 and 29 years. Similar cycles and long-term trends were identified in annual precipitation series, suggesting that high minimum flow values in the past were correctly evaluated, and they need to be considered in estimating T -year minimum specific runoff. This analysis suggests that there is a direct relationship between annual precipitation totals and minimum annual discharges.

In the final section, we constructed series of 7-day minimum specific runoff for the winter-spring and summer-autumn seasons of the hydrological year. Theoretical LP3 lines of non-exceedance of 7-day minimum specific discharges $q_{7d,min}$ for given streams were extrapolated with consideration for lower values to align empirical and theoretical values in the lower part of the graph as closely as possible.

The construction of non-exceedance curves is generally a time-consuming procedure that requires intervention by the author, as it cannot be automated. It is necessary to carefully assess and homogenize the data series where needed especially in the area of low flows, and consider excluding outliers from the series of minimum flows. When searching for parameters of non-exceedance curves, they should be adjusted to minimize errors between empirical values and the theoretical line in the lower part of the curves.

The lowest values of T -year minimum specific runoff are in the Boca stream, while the highest are in the Vajskovský brook. In the Bystrianka stream, T -year minimum specific runoff is lower than in the Štiavnička and Vajskovský brooks, despite having the highest average annual specific runoff.

Winter minima typically occur around January, 20th, while summer minima occur between September, 3–16th. In August, the streams experience the highest water temperatures, which adversely affect the aquatic habitat and survival of species such as the brown trout. Brown trout is widespread in Slovakia in the middle and upper reaches of streams, in mountain streams, and even at high altitudes (up to 1,500 m a.s.l. in the Low Tatras). The main factors affecting its occurrence are water temperature, its cleanliness, and oxygen content.

Spawning occurs in October and November and depends primarily on the water temperature in the respective stream. Larvae hatch naturally from February to March. Therefore, future work will focus on analyzing minimum flows in relation to water temperature.

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