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Encounter probability analysis of irrigation water and reference crop evapotranspiration in irrigation district

Jinping Zhang*, Jiayi Li, Xixi Shi

Institute of Water Resources and Environment, Zhengzhou University, High-tech District, No. 100 Science Road, Zhengzhou City, 450001, Henan Province, China.
* Corresponding author. Tel.: +86-371-60119629. Fax: +86-372-7132666. E-mail: iwhrzhy@sohu.com

Abstract: Based on the data series of the annual reference crop evapotranspiration (ET_0) and the amount of irrigation water (*IR*) from 1970 to 2013 in the Luhun irrigation district, the joint probability distribution of ET_0 and *IR* is established using the Gumbel-Hougaard copula function. Subsequently, the joint probability, the conditional joint probability, and the conditional return period of rich−poor encounter situations of ET_0 and *IR* are analysed. The results show that: (1) For the joint probabilities of rich−poor encounter situations of ET_0 and *IR*, the asynchronous encounter probability is slightly larger than the synchronous encounter probability. (2) When *IR* is in rich state or ET_0 is in poor state, the conditional joint probability is larger, and the conditional return period is smaller. (3) For a certain design frequency of ET_0, if the design frequency decreases, the conditional joint probability of the amount of irrigation water will decrease, therefore the encounter probability of them will decrease. (4) For a certain design frequency of the amount of irrigation water, if the design frequency decreases, the conditional joint probability of ET_0 will increase, thus the encounter probability of them will increase.

Keywords: Luhun irrigation district; Copula function; Reference crop evapotranspiration; Amount of irrigation water; Encounter probability.

INTRODUCTION

The amount of irrigation water (*IR*) is the main factor in irrigation planning and operation management in an irrigation district (Wang et al., 2015; Zhang et al., 2017) and represents the artificial water supply. Reference crop evapotranspiration (ET_0) is an important parameter for calculating the water demand of crops. Moreover, it is the basic data in irrigation planning (Abolfazl et al., 2017; Kong et al., 2013; Li et al., 2007; Liu et al., 2007; Luo et al., 2008; Ni et al., 2006). Irrigation water is affected by precipitation, evaporation, temperature, crop species, and management level (Li et al., 2017; Wang et al., 2015), whereas ET_0 is influenced by meteorological factors and the geographical environment (Fan et al., 2012; Li et al., 2017; Wang et al., 2015). Some scholars have carried out research on the statistical characteristics and evolution laws of these two variables, respectively (Bai et al., 2004; Feng et al., 2011; Huang et al., 2008; Lan et al., 2014; Matin et al., 2016; Ren et al., 2007; Tong and Guo, 2013; Yan et al., 2007). These two variables change randomly, but there is a certain relationship between them. However, the correlation between ET_0 and *IR* has been considered in univariate analysis all the time. Thus, it is difficult to reflect the practical characteristics of ET_0 and the changes of *IR*. ET_0 and *IR* are two mutually influenced hydrological variables, referring to the natural water demand and the artificial water supply in an irrigation district, respectively. Hence, their joint distribution can show the joint statistical characteristics of the natural water demand and the artificial water supply, facilitating the assessment of water shortage risk. However, the characteristics of the joint distribution of these two factors involving their correlation have not been sufficiently studied. Moreover, a study on water supply and demand of ET_0 and (*IR*) has not yet been conducted. Therefore, it is necessary to explore the joint probability, the conditional joint probability as well as the conditional return period of ET_0 and *IR* to evaluate the water supply and demand in an irrigation district.

In this study, based on the data series of the annual ET_0 and *IR* in the Luhun irrigation district of Henan province in China from 1970 to 2013, a two-dimensional joint probability distribution model of ET_0 and *IR* is constructed using the Gumbel-Hougaard copula. This model can be applied to quantitatively analyse the joint probability, the conditional joint probability, and the conditional return period of annual ET_0 and *IR* in various rich−poor encounter situations. Then, the water shortage risk under the artificial water supply condition is studied.

METHODS

Copula functions are used for establishing the joint distribution of multiple random variables, and the marginal distributions of the variates are uniformly distributed on [0, 1]. The Sklar theorem (Song, 2012) is the theoretical basis of a copula function, and its two-dimensional form is expressed as follows: It is assumed that there are two continuous random variables X and Y. Let $F_X(x)$ and $F_Y(y)$ be their marginal distribution functions, and $G(x,y)$ be their joint distribution function. If $F_X(x)$ and $F_Y(y)$ are continuous, there is a unique copula function $C_\theta(u,v)$ defined by:

$$G(x,y)=C_\theta\left(F_X(x),F_Y(y)\right),\forall x,y \tag{1}$$

where $C_\theta(u,v)$ is called the copula function, θ is an undetermined parameter.

In hydrology and water resources, three types of symmetrical Archimedean cluster copula functions that contain only one parameter are widely used (Song, 2012). The correlation measure τ is the Kendall rank correlation coefficient (Davis and Chen, 2007), which describes the nonlinear correlation

Table 1. Three types of common copula functions currently used in hydrological research.

Archimedean copula	$C_\theta(u,v)$	Parameter value	Relation between τ and θ
Frank	$-\frac{1}{\theta}\ln\left[1+\frac{\left(e^{-\theta u}-1\right)\left(e^{-\theta v}-1\right)}{e^{-\theta}-1}\right]$	$\theta\in R$	$\tau=1-\frac{4}{\theta}\left[-\frac{1}{\theta}\int_\theta^0\frac{t}{\exp(t)-1}dt-1\right]$
Clayton	$\left(u^{-\theta}+v^{-\theta}-1\right)^{-1/\theta}$	$\theta>0$	$\tau=\frac{\theta}{\theta+2}$
Gumbel-Hougaard	$\exp\left[-\left((-\ln u)^\theta+(-\ln v)^\theta\right)^{1/\theta}\right]$	$\theta\geq 1$	$\tau=1-\frac{1}{\theta}$

between the variables and has a corresponding relationship with the parameters of the copula function, as shown in Table 1. Based on this, after determining the relationship between the marginal distribution function and the measurement variables, the copula joint distribution functions of the two variables can be obtained by Table 1. Using Kolmogorov-Smirnov (K-S) inspection for performing fitting test and the ordinary least squares (OLS) for evaluating goodness-of-fit, the most suitable copula functions to describe the correlation of the variables are chosen.

PRACTICAL APPLICATION

Research area

The Luhun irrigation district, with a total irrigation area of 1838.48 km^2, is the fourth largest irrigation district in the western Henan province in China. Henan Province is the most populous agricultural province in China. The Luhun irrigation district is located in a hilly area and crosses the Yellow River and the Huai River. The Gross Domestic Product of the Luhun irrigation district is moderate. However, agricultural development lags behind partly because there are three poverty-stricken counties. The irrigation areas are mainly located on both sides of the Yi River (the second tributary of the Yellow River), south of the Luo River and north of the Ru River. The irrigation water in the Luhun irrigation district is from the Luhun reservoir, which is on the upper reaches of the Yi River, and the annual amount of irrigation water is approximately 1.82×10^8 m^3. The irrigation district has a continental monsoon climate, and its annual average rainfall is only 611.02 mm. However, evaporation attains 1034.32 mm, which is due to meteorological drought. Recently, owing to the decrease of the Yi River's runoff, the water storage capacity of the Luhun reservoir has been severely reduced, which greatly influences irrigation activities.

The data series in the Luhun irrigation district involves daily meteorological data and the annual amount of irrigation water from 1970 to 2013. The irrigation water of the Luhun Irrigation District comes from the Luhun Reservoir located on the upper reaches of the Yi River and represents the artificial water supply. The Penman-Monteith formula recommended by FAO in 1998 was used to calculate the annual ET_0 from 1970 to 2013, as shown in Fig. 1.

Joint probability distribution model of ET_0 and *IR*

According to the data series of annual ET_0 (mm) and *IR* (10^8 m^3) in the Luhun irrigation district from 1970 to 2013, the joint probability distribution model was constructed using a copula function. The model expression is given by Eq. (2), and the specific modelling process can be found in Zhang et al. (2017). X and Y represent the frequency of ET_0 and *IR*, respectively. It assumed that u and v are the marginal distribution of annual ET_0 and IR, respectively. Let $F(x,y)$ be their joint probability distribution. The expression of $F(x,y)$ based on the Gumbel−Hougaard copula function is:

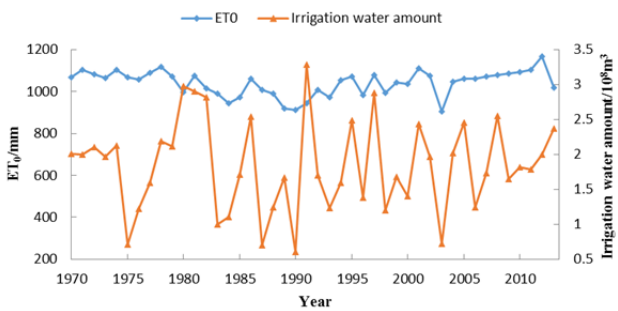

Fig. 1. Data series of ET_0 and IR in the Luhun irrigation district.

$$F(x,y)=\exp\left[-\left((-\ln u)^{1.4205}+(-\ln v)^{1.4205}\right)^{1/1.4205}\right] \quad (2)$$

Application analysis

Joint probability analysis of rich−poor encounter situations

The data series of the annual ET_0 and *IR* in the Luhun irrigation district from 1970 to 2013 obey the GEV and normal distribution, respectively (Zhang et al., 2017). Frequency analysis is commonly used to classify the annual average flow series. Frequency less than 37.5% corresponds to the 'rich' state, and frequency more than 62.5% to the 'poor' state. Frequency between 37.5% and 62.5% is the 'normal' state.

Thus, the values pf = 37.5% and pk = 62.5% are used for classifying a state as 'rich' and 'poor' of the annual ET_0 and *IR* in the Luhun irrigation district, respectively. Then, the established two-dimensional copula joint distribution model is used to analyse various rich−poor encounter situations.

The numerical division of the annual ET_0 and *IR* according to the frequency distribution curve is shown in Table 2.

Table 2. Classification standards of rich−poor states between ET_0 and *IR*.

Variables \ Frequency	37.5%	62.5%
ET_0 (mm)	1071.9	1035.1
Irrigation water (10^8 m^3)	2.05	1.61

The frequency method is used to classify the rich−poor encounter situations of the annual ET_0 and *IR* into nine types that are further classified as synchronous and asynchronous as follows:

Rich−rich: $p_1 = P\left(X \ge x_{pf}, Y \ge y_{pf}\right)$;

Rich−normal: $p_2 = P\left(X \ge x_{pf}, y_{pk} < Y < y_{pf}\right)$;

Rich−poor: $p_3 = P\left(X \ge x_{pf}, Y \le y_{pk}\right)$;

Normal−rich: $p_4 = P\left(x_{pk} < X < x_{pf}, Y \ge y_{pf}\right)$;

Normal−normal: $p_5 = P\left(x_{pk} < X < x_{pf}, y_{pk} < Y < y_{pf}\right)$;

Normal−poor: $p_6 = P\left(x_{pk} < X < x_{pf}, Y \le y_{pk}\right)$;

Poor−rich: $p_7 = P\left(X \le x_{pk}, Y \ge y_{pf}\right)$;

Poor−normal: $p_8 = P\left(X \le x_{pk}, y_{pk} < Y < y_{pf}\right)$;

Poor−poor: $p_9 = P\left(X \le x_{pk}, Y \le y_{pk}\right)$.

After the calculation, the results of the rich−poor encounter analysis of the annual ET_0 and *IR* in the Luhun irrigation district are shown in Table 3.

Table 3 shows the following:

(1) The encounter probability of 'Rich−rich' is the highest among these nine situations, which is up to 21.45%.

(2) Among the synchronous probabilities of ET_0 and *IR*, there are few differences between the probabilities of 'rich−rich' and 'poor−poor' (21.45% and 20.14%, respectively), and 'Normal−normal' is the lowest (7.91%).

(3) Among the asynchronous probabilities of ET_0 and IR, the encounter probability of 'Rich−normal' (or 'Normal−rich') is the lowest among these nine situations, which is 7.89%; the encounter probability of 'Rich−poor' (or 'Poor−rich') is 8.16%; the encounter probability of 'Normal−poor' (or 'Poor−normal') is 9.20%. There are few differences among these six types of asynchronous probabilities.

(4) In total, the asynchronous probabilities of ET_0 and *IR* are higher than synchronous probabilities, which are 50.50% and 49.50%, respectively, but their differences are not significant.

Encounter probability analysis of conditional joint probability (CJP) and conditional return period (CRP) in rich and poor state

When ET_0 is in rich state $X \ge x_{pf}$, normal state $x_{pk} \le X \le x_{pf}$, or poor state $X \le x_{pk}$, the CJP and the corresponding CRP of *IR* not exceeding a given value are given by Eq. (3) and Eq. (4), respectively.

$$\begin{cases} F_{Y_i|x_f}\left(X, Y_i\right) = P\left(Y_i \le y \middle| X \ge x_{pf}\right) = \dfrac{F_Y\left(y\right) - F\left(x_{pf}, y\right)}{1 - F_X\left(x_{pf}\right)} \\ F_{Y_i|x_p}\left(X, Y_i\right) = P\left(Y_i \le y \middle| x_{pk} \le X \le x_{pf}\right) = \dfrac{F\left(x_{pf}, y\right) - F\left(x_{pk}, y\right)}{F_X\left(x_{pf}\right) - F_X\left(x_{pk}\right)} \\ F_{Y_i|x_k}\left(X, Y_i\right) = P\left(Y_i \le y \middle| X \le x_{pk}\right) = \dfrac{F\left(x_{pk}, y\right)}{F_X\left(x_{pk}\right)} \end{cases} \tag{3}$$

$$T_{Y_i|x_f}\left(X, Y_i\right) = \frac{1}{F_{Y_i|x_f}\left(X, Y_i\right)}, T_{Y_i|x_p}\left(X, Y_i\right) = \\ = \frac{1}{F_{Y_i|x_p}\left(X, Y_i\right)}, T_{Y_i|x_k}\left(X, Y_i\right) = \frac{1}{F_{Y_i|x_k}\left(X, Y_i\right)} \tag{4}$$

where $F_{Y|x}(\bullet)$ represents the CJP of *IR* and $T_{Y|x}(\bullet)$ represents its corresponding CRP.

When *IR* is in rich sate $Y \ge y_{pf}$, normal state $y_{pk} \le Y \le y_{pf}$, or poor state $Y \le y_{pk}$, the CJP and the corresponding CRP of ET_0 exceeding a given value are given by Eq. (5) and Eq. (6), respectively.

$$\begin{cases} F_{X_i|y_f}\left(X_i, Y\right) = P\left(X_i \ge x \middle| Y \ge y_{pf}\right) = \\ = \dfrac{1 - F_Y\left(y_{pf}\right) - F_X\left(x\right) + F\left(x, y_{pf}\right)}{1 - F_Y\left(y_{pf}\right)} \\ F_{X_i|y_p}\left(X_i, Y\right) = P\left(X_i \ge x \middle| y_{pk} \le Y \le y_{pf}\right) = \\ = \dfrac{F_Y\left(y_{pf}\right) - F_Y\left(y_{pk}\right) - F\left(x, y_{pf}\right) + F\left(x, y_{pk}\right)}{F_Y\left(y_{pf}\right) - F_Y\left(y_{pk}\right)} \\ F_{X_i|y_k}\left(X_i, Y\right) = P\left(X_i \ge x \middle| Y \le y_{pk}\right) = \\ = \dfrac{F_Y\left(y_{pk}\right) - F\left(x, y_{pk}\right)}{F_Y\left(y_{pk}\right)} \end{cases} \tag{5}$$

$$T_{X_i|y_f}\left(X_i, Y\right) = \frac{1}{F_{X_i|y_f}\left(X_i, Y\right)}, \\ T_{X_i|y_p}\left(X_i, Y\right) = \frac{1}{F_{X_i|y_p}\left(X_i, Y\right)} \\ T_{X_i|y_k}\left(X_i, Y\right) = \frac{1}{F_{X_i|y_k}\left(X_i, Y\right)} \tag{6}$$

where $F_{X|y}(\bullet)$ represents the CJP of ET_0, and $T_{X|y}(\bullet)$ represents its corresponding CRP.

Table 3. Encounter probabilities (%) of synchronous−asynchronous encounter between ET_0 and *IR*.

Synchronous probabilities				Asynchronous probabilities						
Rich−rich	Normal−normal	Poor−poor	Total	Rich−normal	Rich−poor	Normal−poor	Normal−rich	Poor−rich	Poor−normal	Total
21.45	7.91	20.14	49.50	7.89	8.16	9.20	7.89	8.16	9.20	50.50

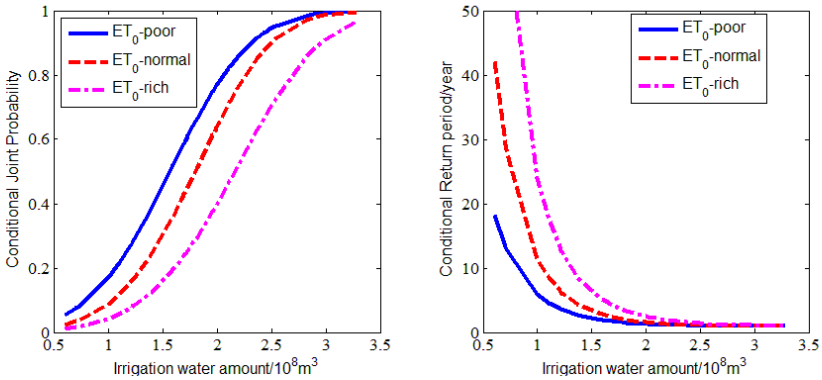

Fig. 2. CJP and CRP of *IR* not exceeding a given value.

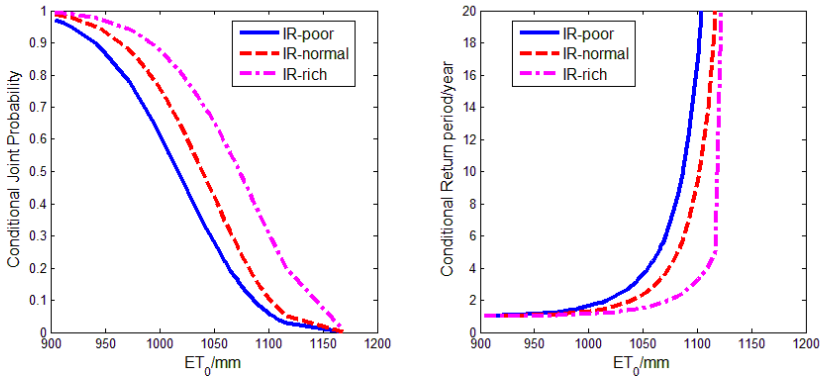

Fig. 3. CJP and CRP of ET_0 exceeding a given value.

Fig. 2 shows the values of CJP and CRP of *IR* not exceeding a given value when ET_0 is in rich, normal, and poor state. The results are as follows:

(1) When *IR* increases, the CJP of *IR* increases, whereas the CRP of *IR* decreases.

(2) When ET_0 is in rich state ($ET_0 \geq 1071.9$ mm), the CJP of *IR* is minimum, whereas the CRP of *IR* is maximum. By contrast, when ET_0 is in poor state ($ET_0 \leq 1035.1$ mm), the CJP and the CRP of *IR* exhibit the inverse results: the CJP is maximum, whereas the CRP is minimum.

(3) When ET_0 is in rich state ($ET_0 \geq 1071.9$ mm), the CJP when *IR* is less than 1.0×10^8, 2.0×10^8, and 3.0×10^8 m^3 are approximately 0.04, 0.40, and 0.91, respectively. The corresponding values of the CRP are approximately 23.87, 2.51, and 1.10 years.

(4) When ET_0 is in normal state (1035.1 mm < ET_0 < 1071.9 mm), the CJP when *IR* is less than 1.0×10^8, 2.0×10^8, and 3.0×10^8 m^3 are approximately 0.09, 0.63, and 0.98, respectively. The corresponding values of the CRP are approximately 11.3, 1.54, and 1.02 years.

(5) When ET_0 is in poor state ($ET_0 \leq 1035.1$ mm), the CJP when *IR* is less than 1.0×10^8, 2.0×10^8, and 3.0×10^8 m^3 are approximately 0.17, 0.77, and 0.99, respectively. The corresponding values of the CRP are approximately 5.86, 1.23, and 1.01 years.

From (3) and (4), it can be concluded that when ET_0 is in rich and normal state and *IR* is in normal and poor state, the probabilities that the artificial water supply cannot meet water demand are small. However, from (5) we can know that when ET_0 is in poor state and *IR* is in rich state, the probability is up to 0.99, which means that there may be a phenomenon of wasting water resources in the irrigation district. This information may facilitate irrigation regulation.

Fig. 3 shows the value of CJP and CRP of ET_0 exceeding a given value when *IR* is in rich, normal, and poor state. The results are as follows:

(1) When ET_0 increases, the CJP of ET_0 decreases, whereas the CRP of ET_0 increases.

(2) When *IR* is in rich state ($IR \geq 2.05\times10^8$ m^3), the CJP of ET_0 is maximum, whereas the CRP of ET_0 is minimum; by contrast, when *IR* is in poor state ($IR \leq 1.61\times10^8$ m^3), the CJP and CRP of ET_0 exhibit the inverse results: the CJP is minimum, whereas the CRP is maximum.

(3) When *IR* is in rich state ($IR \geq 2.05\times10^8$ m^3), the CJP when ET_0 is greater than 950 mm, 1050 mm, and 1150 mm are approximately 0.97, 0.65, and 0.08, respectively. The corresponding values of the CRP are approximately 1, 1.48, and 12.5 years.

(4) When *IR* is in normal state (1.61×10^8 m^3 < *IR* < 2.05×10^8 m^3), the CJP when ET_0 is greater than 950 mm, 1050 mm and 1150 mm are approximately 0.93, 0.43, and 0.04, respectively. The corresponding values of the CRP are approximately 1.02, 2.23, and 25 years.

(5) When *IR* is in poor state ($IR \leq 1.61\times10^8$ m^3), the CJP when ET_0 is greater than 950 mm, 1050 mm and 1150 mm are approximately 0.87, 0.28, and 0.02, respectively. The corresponding values of the CRP are approximately 1.15, 3.57, and 50 years.

From (4) and (5), it can be concluded that when *IR* is in normal and poor state and ET_0 is in rich and normal state the probabilities that the artificial water supply cannot meet water demand are small, which means that it is less likely to be uncoordinated of water supply and demand. However, from (3) we can know that when *IR* is in rich state and ET_0 is in poor state

the probability is up to 0.97, which means that water using may be not reasonable in the irrigation district. Thus, the water supply cannot meet demand in the low frequency and long return period.

Analysis of the CJP and CRP with a certain design frequency

The smaller CJP and longer CRP of either *IR* or ET_0 indicates that the artificial water supply can mostly satisfy the water demand, but when the extremely water supply−demand situations occur, whether water supply−demand is coordinated is very important. Thus, the CJP and CRP of *IR* (or ET_0) with a certain design frequency of ET_0 (or *IR*) need to be considered as follows:

Condition I:

$$\begin{cases} P(Y \le y | X \ge x) = \dfrac{F_Y(y) - F(x,y)}{1 - F_X(x)} \\ T(Y \le y | X \ge x) = \dfrac{1}{P(Y \le y | X \ge x)} \end{cases} \tag{7}$$

where $P(Y \le y | X \ge x)$ is the CJP of *IR* in poor state not exceeding a certain design frequency when ET_0 in rich state exceeds a specific frequency, and $T(Y \le y | X \ge x)$ is the corresponding CRP.

Condition II:

$$\begin{cases} P(X \ge x | Y \le y) = \dfrac{F_Y(y) - F(x,y)}{F_Y(y)} \\ T(X \ge x | Y \le y) = \dfrac{1}{P(X \ge x | Y \le y)} \end{cases} \tag{8}$$

where $P(X \ge x | Y \le y)$ is the CJP of ET_0 in rich state exceeding a certain design frequency when *IR* in poor state does not exceed a specific frequency, and $T(X \ge x | Y \le y)$ is the corresponding CRP.

These two types of CJP and CRP reflect the probability of unbalanced phenomenon of artificial water supply and demand, and reveal the corresponding water resources utilization. The results of the CJP and the CRP are shown in Tables 4 and 5, respectively.

Table 4 shows that when the frequency of ET_0 in rich state exceeds a certain design frequency, the CJP of *IR* decreases as the design frequency of ET_0 decreases, and the encounter probability of these two variables decreases. The results are as follows:

(1) When the frequency of ET_0 in rich state exceeds a certain design frequency, the probability of *IR* being in extremely poor state without exceeding the design frequency of 95% is less than 0.015, and the CRP is considerably long (the minimum is 66.67 years, the maximum is 200 years). Thus, in such an extreme situation, irrigation water cannot meet demand within 66.67−200 years. Therefore, the probability of unbalanced phenomenon of water supply and demand in irrigation district is low.

(2) When the frequency of ET_0 in rich state exceeds a certain design frequency, the probability of *IR* being in poorer state without exceeding the design frequency of 90% and 75% is less than 0.13 (respectively, 0.015−0.040, 0.046−0.126), and the CRP is longer (respectively, 25.00−66.67 years, 7.94−21.74 years).

(3) When the frequency of ET_0 in rich state exceeds a certain design frequency, the probability of *IR* being in poor state without exceeding the design frequency of 62.5% is less than 0.20. In addition, the maximum CRP is 13.89 years, and the minimum is 5.13 years.

Thus, it can be concluded from Table 4 that the probability of severe water shortage is low when the frequency of water demand exceeds a certain design frequency in the irrigation district.

Table 5 shows that when *IR* in poor state does not exceed a certain design frequency, the CJP of ET_0 increases as the design frequency of *IR* decreases, and the encounter probability of these two variables increases. The results are as follows:

(1) When the frequency of *IR* in poor state does not exceed the design frequency, the probability of ET_0 in extremely rich state exceeding the design frequency of 5% is less than 0.033, and the CRP is considerably long (the minimum is 30.30 years, the maximum is 47.62 years).

(2) When the frequency of *IR* does not exceed the design frequency, the probability of ET_0 in richer state exceeding the design frequency of 25% and 10% is less than 0.122, and the CRP is longer (respectively, 8.20−12.20 years, 18.52−29.41 years).

(3) When the frequency of *IR* does not exceed the design frequency, the probability of ET_0 in rich state exceeding the design frequency of 37.5% is less than 0.161, and the average CRP is 7.78 years.

Table 4. CJP I and CRP I of *IR* when ET_0 in rich state exceeds a certain design frequency.

		IR (10^8 m^3)	0.7	1.0	1.4	1.6			*IR* (10^8 m^3)	0.7	1.0	1.4	1.6
	ET_0 (mm)	P%	95	90	75	62.5		ET_0 (mm)	P%	95	90	75	62.5
CJP I	1071.9	37.5	0.015	0.040	0.126	0.195	CRP I	1071.9	37.5	66.67	25.00	7.94	5.13
	1079.2	25	0.012	0.033	0.100	0.156		1079.2	25	83.33	30.30	10.00	6.41
	1102.7	10	0.007	0.020	0.063	0.099		1102.7	10	142.86	50.00	15.87	10.10
	1113.2	5	0.005	0.015	0.046	0.072		1113.2	5	200.00	66.67	21.74	13.89

Table 5. CJP II and CRP II of ET_0 when *IR* in poor state does not exceed a certain design frequency.

		ET_0 (mm)	1071.9	1079.2	1102.7	1113.2			ET_0 (mm)	1071.9	1079.2	1102.7	1113.2
	IR (10^8 m^3)	P%	37.5	25	10	5		*IR* (10^8 m^3)	P%	37.5	25	10	5
CJP II	0.7	95	0.107	0.082	0.034	0.021	CRP II	0.7	95	9.35	12.20	29.41	47.62
	1.0	90	0.117	0.090	0.038	0.024		1.0	90	8.55	11.11	26.32	41.67
	1.4	75	0.143	0.107	0.047	0.029		1.4	75	6.99	9.35	21.28	34.48
	1.6	62.5	0.161	0.122	0.054	0.033		1.6	62.5	6.21	8.20	18.52	30.30

From the analysis of Table 4 and 5, the CRP of *IR* in poor state without exceeding the design frequency when the frequency of ET_0 in rich state exceeds a certain design frequency, and the CRP of ET_0 in rich state exceeding the design frequency when the frequency of *IR* in poor state does not exceed the design frequency are considerably long, which means that the probability of the extremely water supply−demand occurring in the irrigation water system is very low. At the same time, the analysis indicates that water supply and demand in the irrigation district is relatively coordinated.

CONCLUSION

Using Gumbel-Hougaard copula function, the joint distribution model of the amount of irrigation water and reference crop evapotranspiration was constructed. The frequency of rich−poor encounter situations, the CJP, and the CRP of the amount of irrigation water and reference crop evapotranspiration were analysed to study the unbalanced phenomena in artificial water supply and demand conditions. The following conclusions can be concluded:

(1) The asynchronous frequency was found to be slightly larger than the synchronous frequency in the Luhun irrigation district. Thus, water supply and demand of irrigation district may be in high risk of incoordination, but the probability of extremely uncoordinated artificial water supply and demand (rich *IR*, poor ET_0 or rich ET_0, poor *IR*) is not high.

(2) Using the established joint probability distribution of ET_0 and *IR* for a specific ET_0 or *IR*, the encounter probability of artificial water supply and demand was estimated. Similarly, the probability of water shortage exceeding ET_0 or not exceeding *IR* was also obtained. Thus, the water supply and demand, and utilization of water resources in irrigation district could be described and evaluated quantitatively.

(3) Regardless of the design frequency of reference crop evapotranspiration or the amount of irrigation water, severe water supply−demand issues would not occur in the irrigation district. This is of great significance for the development of the irrigation system, the optimal allocation of regional water resources, and other practical problems in the irrigation district.

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