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Performance of a shallow-water model for simulating flow over trapezoidal broad-crested weirs

Jaromír Říha[1*], David Duchan[1], Zbyněk Zachoval[1], Sébastien Erpicum[2], Pierre Archambeau[2], Michel Pirotton[2], Benjamin Dewals[2]

[1] Faculty of Civil Engineering, Brno University of Technology, Brno, Czech Republic.
[2] Hydraulics in Environmental and Civil Engineering (HECE), Research unit Urban & Environmental Engineering, University of Liege (ULiège), Belgium.
[*] Corresponding author. E-mail: riha.j@fce.vutbr.cz

Abstract: Shallow-water models are standard for simulating flow in river systems during floods, including in the near-field of sudden changes in the topography, where vertical flow contraction occurs such as in case of channel overbanking, side spillways or levee overtopping. In the case of stagnant inundation and for frontal flow, the flow configurations are close to the flow over a broad-crested weir with the trapezoidal profile in the flow direction (i.e. inclined upstream and downstream slopes). In this study, results of shallow-water numerical modelling were compared with seven sets of previous experimental observations of flow over a frontal broad-crested weir, to assess the effect of vertical contraction and surface roughness on the accuracy of the computational results. Three different upstream slopes of the broad-crested weir (V:H = 1:Z_1 = 1:1, 1:2, 1:3) and three roughness scenarios were tested. The results indicate that, for smooth surface, numerical simulations overestimate by about 2 to 5% the weir discharge coefficient. In case of a rough surface, the difference between computations and observations reach up to 10%, for high relative roughness. When taking into account mentioned the differences, the shallow-water model may be applied for a range of engineering purposes.

Keywords: Discharge coefficient; Frontal broad-crested weir; Shallow flow modelling; Rough weir crest.

INTRODUCTION

The hydraulics of trapezoidal broad-crested weirs (i.e. with inclined upstream and downstream slopes) is of high engineering relevance. Trapezoidal broad-crested weirs are not only common hydraulic engineering structures (hydropower, discharge measurement, etc.) but topographic features similar to both frontal, side or oblique broad-crested weirs are also observed in diverse flow configurations, such as in case of channel overbanking or when dikes (or levees) are overtopped during flood events (Fig. 1).

Fig. 1. Levee overtopping, to some extent similar to the flow over a broad-crested weir.

The hydraulics of broad-crested weirs was investigated in many experimental studies, such as Felder and Chanson (2012), Goodarzi et al. (2012), Haddadi and Rahimpour (2012), Hager and Schwalt (1994), Madadi, et al. (2013, 2014). Several of these studies highlighted the considerable influence of the weir upstream slope on the discharge coefficient. As summarized in Table 1, existing research covers a range of values of the weir upstream slope. For embankment weirs with equal upstream and downstream slopes of V:H = 1:2, Fritz and Hager (1998) determined the discharge coefficient as a function of the relative crest length, considering multiple crest configurations including the case of the broad-crested weir. Major (2013) evaluated the discharge coefficient for an upstream slope of V:H = 1:1. Sargison and Percy (2009) and Goodarzi et al. (2012) provide discharge coefficient values for smooth trapezoidal broad-crested weirs with various upstream and downstream slopes. Similarly, values of the discharge coefficient derived from extensive research are summarized in the international standard for discharge measurement ISO 4362 (1999). Tokyay and Altan-Sakarya (2011) analyzed the local head loss at positive and negative steps considering either an abrupt step or a V:H = 1:1 slope. They related the local head loss to the Froude number and the relative step height. Madadi et al. (2014) identified increased discharge coefficient up to 10% with decreasing the upstream face slope from 90° to 21°.

Pařílková et al. (2012) investigated experimentally the influence of the weir surface roughness on the discharge coefficient. For vegetated weir surface conventional flow resistance equations may also be used (Gualtieri et al., 2018).

Computational predictions of the flow profiles and discharge coefficient for overflooded obstacles and broad-crested weirs can be obtained using three-dimensional or vertical-two-dimensional models (Velísková et al., 2018). These models are based either on Eulerian methods (Hargreaves et al., 2007; Haun et al., 2011; Kirkgoz et al., 2008; Sarker and Rhodes 2004) or on meshless techniques (Xu and Jin, 2017). Depth-averaged models have also been developed to account for the strong vertical flow contraction at broad-crested weirs and the

Table 1. Selected experimental research on flow over broad-crested weirs, with various upstream slopes (V:H = 1:Z_1, see Fig. 2).

Upstream slope Z_1	$Z_1 = 1$	$Z_1 = 2$	$Z_1 = 3$
Fritz and Hager (1998)		✓	
ISO 4362 (1999)	✓	✓	✓
Zerihun and Fenton (2007)		✓	
Sargison and Percy (2009)	✓	✓	
Tokyay and Altan-Sakarya (2011)	✓		
Goodarzi et al. (2012)	✓	✓	
Major (2013)	✓		

resulting non-hydrostatic effects, leading to a good agreement between the numerical results and experimental observations (Darvishi et al., 2017). For a range of research applications, similar models were used successfully for reproducing the hydraulics of embankment breaching (Cantero-Chinchilla et al., 2018; Castro-Orgaz and Hager, 2013).

As shown in Fig. 1, flow conditions similar to flow over broad-crested weirs are also observed in multiple situations, such as in inundation flow, for which standard shallow-water models are routinely applied (Gallegos et al. 2009; Horritt and Bates, 2001). In principle, shallow-water flow models fail to reproduce flow processes at vertical flow contractions, which may lead to errors in the numerical estimation of the overflowing discharge or the upstream head. Nonetheless, no systematic quantification of this error exists so far for various upstream and downstream weir slopes, weir roughness and a broad range of discharges. The objective of the present paper is to fill this gap by assessing systematically the performance of an operational shallow-water flow model applied to the computation of trapezoidal broad-crested weir flow with three different upstream slopes, 8 flow discharges and 3 weir roughness scenarios. The computational results are compared against existing experimental datasets (Fritz and Hager, 1998; Goodarzi et al., 2012; Major, 2013; Sargison and Percy, 2009; Tokyay and Altan-Sakarya, 2011; Zerihun and Fenton, 2007) and standards (ISO 4362, 1999). Madadi et al. (2014) tested different upstream slopes (V:H = 1:0, 1:0.29, 1:0.73, 1:1.19, 1:2.61). Therefore their experimentally determined discharge coefficients were interpolated for the slopes V:H = 1:1 and 1:2 and then compared against numerical results.

METHODS

In total 66 numerical simulations were conducted to compare computed values of the discharge coefficient C_D to existing experimental data and to a standard.

Numerical simulations

We used the numerical model WOLF developed at the University of Liege and routinely applied for research and engineering purposes (e.g., Stilmant et al., 2018). It solves the shallow-water equations, written in conservative form, by means of a finite volume scheme with a flux-vector splitting developed in-house, which is well-balanced with respect to the discretization of the pressure and bottom slope terms (Erpicum et al., 2010). The time integration is performed using a 2-step Runge–Kutta algorithm and a semi-implicit treatment of the bottom friction term is used. The time step is adaptive and computed based on the Courant–Friedrichs–Lewy (CFL) stability criterion, with a CFL number equal to 0.25. It takes values of the order of 10^{-3} s. More details on the model equations and numerical scheme are presented by Erpicum et al. (2010).

The computational domain represents a channel, with a horizontal bottom, in which various trapezoidal broad-crested weir geometries are inserted. It was discretized using a regular grid with spacing $\Delta x = 0.01$ m. Since we consider frontal flow with respect to the weir (no crosswise velocity), the width of the computational domain was limited to a single cell (of width $\Delta y = 0.01$ m); but no lateral friction was imposed. This corresponds to a vertical "slice" in a very wide channel.

The geometry of the trapezoidal broad-crested weir is sketched in Fig. 2. The numerical simulations were performed for an upstream weir height $P = 0.5$ m and a crest width $t = 0.5$ m. The upstream and downstream slopes (1:Z_1 and 1:Z_2) were varied between 1:1 and 1:3. With the considered grid spacing $\Delta x = 0.01$ m, all weir edges were exactly fitting the weir geometry.

The upstream boundary condition for the simulations was a prescribed specific discharge q at the entrance cross-section, distant 8.5 m from the upstream crest edge. The downstream end of the computational domain corresponds to the toe of the downstream slope, where free outflow was prescribed as downstream boundary condition. As the flow regime changes from subcritical to supercritical approximately at the downstream crest edge of the weir, this boundary is not modified when the weir geometry is varied. The side walls were considered as smooth impermeable boundaries.

Computations were performed for both smooth and rough weir surfaces:

- For smooth weirs, a Manning roughness coefficient $n = 0.01$ s/m$^{1/3}$ was applied (this corresponds to a Nikuradse roughness size of $k_s < 0.0002$ m), and the specific discharge was varied in-between $q = 0.02$ m^2/s and $q = 0.15$ m^2/s. The full range of upstream slopes was considered (i.e. 1:Z_1 varying between 1:1 and 1:3).
- The effect of the weir crest roughness on the discharge coefficient was studied for the set of Nikuradse roughness size $k_s = 0.014$, 0.020 and 0.024 m, for specific discharges varying from $q = 0.005$ to $q = 0.25$ m^2/s, and an upstream slope $Z_1 = 2.5$, consistently with the study of Pařílková et al. (2012).

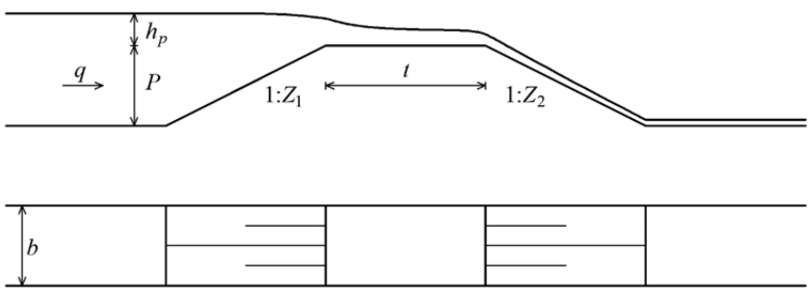

Fig. 2. Sketch of the flow over a trapezoidal broad-crested weir, with main notations.

Reference data

Following ISO 4362 (1999), the specific discharge q passing over a frontal trapezoidal broad-crested weir with a rectangular cross-section may be computed based on the following equation:

$$q = \left(\frac{2}{3}\right)^{3/2} C_D C_v \sqrt{g} h_p^{3/2}, \tag{1}$$

where g is the acceleration of gravity and h_p the overflow head (Fig. 2). C_D represents the discharge coefficient and C_v the approach velocity coefficient calculated iteratively from this expression (ISO 4362, 1999):

$$C_v = \left\{1 + \frac{4}{27} C_v^2 \left[C_D \frac{h_p}{\left(h_p + P\right)}\right]^2\right\}^{3/2}, \tag{2}$$

where P is the upstream weir height. Dimensional analysis shows that the discharge coefficient C_D depends on seven dimensionless parameters:

$$C_D = f\left(\mathrm{R}, \mathrm{W}, \frac{h_p}{t}, \frac{h_p}{P}, \frac{h_p}{k_s}, \frac{1}{Z_1}, \frac{1}{Z_2}\right), \tag{3}$$

where R is the Reynolds number, W the Weber number, k_s the Nikuradse equivalent roughness size, $1{:}Z_1$ and $1{:}Z_2$ the upstream and downstream slopes, t the crest width (in the streamwise direction).

According to ISO 4362 (1999), the upstream slope $1{:}Z_2$ does not influence the discharge coefficient C_D when the overflow head remains small enough compared to the crest width: $h_p/t \leq 0.5$. Moreover, the influence of R and W vanishes for $h_p \geq 0.05$ m, $P \geq 0.15$ m and channel width $b \geq 0.3$ m. Under such conditions and for a smooth weir surface (C_D is not influenced by h_p/k_s), Eq. (3) reduces to:

$$C_D = f\left(\frac{h_p}{t}, \frac{h_p}{P}, \frac{1}{Z_1}\right), \tag{4}$$

stating that C_D is controlled by the relative crest width (h_p/t), the relative weir height (h_p/P) and the upstream slope ($1{:}Z_1$). ISO 4362 (1999) does not document the influence of varying weir height; therefore, this was also not examined here. ISO 4362 (1999) specifies an accuracy of ±0.5% for the reported values of C_D.

RESULTS AND DISCUSSION

Smooth surface

For each configuration and each prescribed inflow discharge, the hydraulic head upstream of the weir was deduced from the computed water depth in the upstream part of the simulation domain (Table 2). Next, corresponding discharge coefficients were evaluated using Eqs. (1) and (2), in Tables 3 and 4.

Table 2. Computed overflow heads h_p as a function of the specific discharge q and the upstream slope Z_1.

Upstream slope Z_1	Specific discharge q (m^2/s)							
	0.02	0.03	0.05	0.07	0.09	0.11	0.13	0.15
1	0.0537	0.0698	0.0972	0.1209	0.1423	0.1622	0.1807	0.1983
2	0.0532	0.0692	0.0965	0.1202	0.1415	0.1613	0.1798	0.1973
3	0.0530	0.0690	0.0962	0.1199	0.1412	0.1610	0.1795	0.1970

Table 3. Computed discharge coefficients C_D for various specific discharges and upstream slopes.

Specific discharge q (m^2/s)	Upstream slope Z_1 = 1		Upstream slope Z_1 = 2		Upstream slope Z_1 = 3	
	h_p/t	Computed C_D	h_p/t	Computed C_D	h_p/t	Computed C_D
0.02	0.1074	0.941	0.1064	0.954	0.1060	0.960
0.03	0.1395	0.952	0.1384	0.964	0.1379	0.968
0.05	0.1943	0.963	0.1930	0.973	0.1924	0.977
0.07	0.2418	0.969	0.2403	0.978	0.2397	0.981
0.09	0.2847	0.973	0.2831	0.981	0.2825	0.984
0.11	0.3243	0.975	0.3226	0.983	0.3219	0.986
0.13	0.3614	0.977	0.3596	0.985	0.3589	0.987
0.15	0.3965	0.979	0.3947	0.986	0.3940	0.989

Table 4. Comparison of discharge coefficients C_D for various upstream weir slopes (see Fig. 3).

Upstream slope Z_1 = 1			Upstream slope Z_1 = 2			Upstream slope Z_1 = 3		
C_D from ISO 4362 (1999)	Computed C_D (WOLF)	Relative difference	C_D from ISO 4362 (1999)	Computed C_D (WOLF)	Relative difference	C_D from ISO 4362 (1999)	Computed C_D (WOLF)	Relative difference
0.909	0.941	3.5 %	0.937	0.954	1.8 %	0.947	0.960	1.4 %
0.913	0.952	4.3 %	0.942	0.964	2.3 %	0.952	0.968	1.7 %
0.919	0.963	4.8 %	0.951	0.973	2.3 %	0.962	0.977	1.6 %
0.923	0.969	5.0 %	0.957	0.978	2.2 %	0.967	0.981	1.4 %
0.927	0.973	5.0 %	0.962	0.981	2.0 %	0.972	0.984	1.2 %
0.930	0.975	4.8 %	0.966	0.983	1.8 %	0.976	0.986	1.0 %
0.934	0.977	4.6 %	0.970	0.985	1.5 %	0.980	0.987	0.7 %
0.938	0.979	4.4 %	0.973	0.986	1.3 %	0.983	0.989	0.6 %

The numerical simulations confirmed that downstream slope Z_2 has no influence on the overflow head and the discharge coefficient for the broad-crested weir conditions, which agrees with ISO 4362 (1999) and also with data published by Sargison and Percy (2009). Therefore the results of simulations with varying downstream slopes are not further detailed hereafter.

Comparison against ISO 4362 (1999)

In Fig. 3, the computed discharge coefficients C_D are plotted against the values recommended by the standard ISO 4362 (1999) for the range $0.10 \leq h_p/t \leq 0.40$ and for the case of smooth weir surface. Compared to the standard, the numerical predictions systematically overestimate the discharge coefficient, by up to 5%. The deviation between the standard and the numerical predictions is strongly influenced by the weir upstream slope: the steeper the upstream slope, the stronger the deviation. Indeed, the overestimation of C_D is in the ranges 1 to 2%, 1 to 3% and 3 to 6% for upstream slopes of $Z_1 = 3$, $Z_1 = 2$ and $Z_1 = 1$, respectively.

Computed discharge coefficients vs. earlier experimental data

In Figs. 4 to 6, the discharge coefficients evaluated from our simulations are compared not only with the values of the standard ISO 4362 (1999), but also with experimental data from Fritz and Hager (1998), Zerihun and Fenton (2007), Sargison and Percy (2009), Tokyay and Altan-Sakarya (2011), Goodarzi et al. (2012), Major (2013) and Madadi et al. (2014).

The graphs reveal substantial differences between the individual data sources. For instance, Goodarzi et al. (2012) found C_D values more than 10% higher than all other sources. For an upstream slope $Z_1 = 1$, the results by Toykay and Altan-Sakarya (2011), Major (2013) and by Madadi et al. (2014) agree relatively well with the values of ISO 4362 (1999), whereas the discharge coefficients estimated by Sargison and Percy (2009) are by 5 to 7% lower, with a stronger difference for smaller values of h_p/t.

For the slope $Z_1 = 2$ the values of C_D derived by Fritz and Hager (1998) exceed by about 2% the values recommended by ISO 4362 (1999). The only one C_D value published by Zerihun and Fenton (2007) provide only minor difference when compared with ISO 4362 (1999). In contrast, Sargison and Percy (2009) and Madadi et al. (2014) provide discharge coefficients by 4 to 8% smaller than those recommended by ISO 4362 (1999) for the same configuration, better agreement provide data interpolated from Madadi et al. (2014).

In all cases except for data of Goodarzi et al. (2012) the discharge coefficients from numerical simulations exceed the experimental values. The best agreement for the upstream slope $Z_1 = 2$ is obtained with data by Fritz and Hager (1988) and one value of C_D derived by Sargison and Percy (2009) for $h_p/t = 0.13$. The higher discharges coefficients obtained from the simulations may be attributed to limitations inherent to the shallow-water model, namely the assumption of hydrostatic pressure distribution and missing the effect of rapid vertical contractions which may lead to detached flow on the upstream part of the weir crest.

Rough weir crest

In Fig. 7 the results of simulations considering a rough surface for the weir crest are compared with the results of previous measurements (Pařílková et al., 2012).

When a rough surface of the weir crest is taken into account, the shallow-water flow simulations overestimate again C_D compared to experimental observations, particularly for values of h_c/k_s in between 0.5 and 3.5, i.e. for relatively high roughness heights. The differences between computations and observations are in the range of 5% to 8%. When the ratio h_c/k_s is increased, the agreement between simulations and measured values significantly improves, especially for values of h_c/k_s above 5. h_c is critical depth.

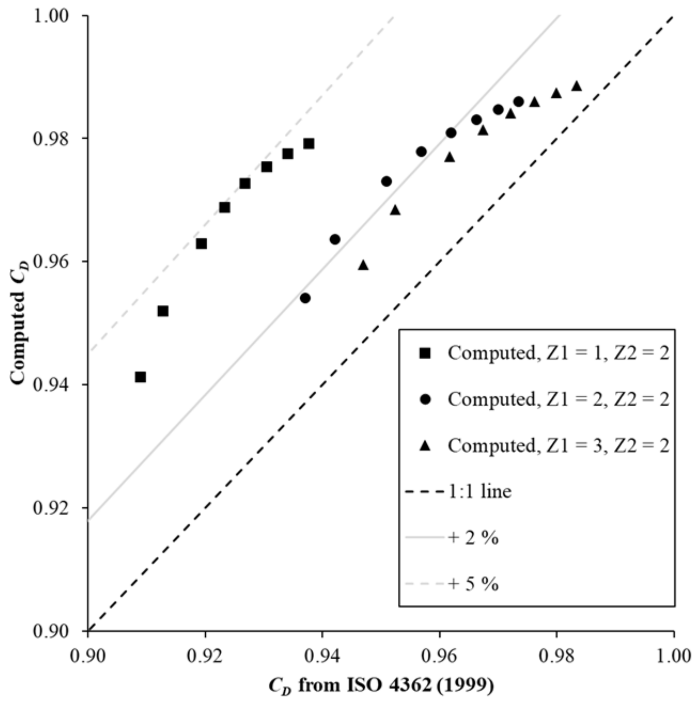

Fig. 3. Discharge coefficients C_D computed with the shallow-water model vs. recommended by ISO 4362 (1999), for various upstream slopes ($1:Z_1$) and downstream slopes ($1:Z_2$).

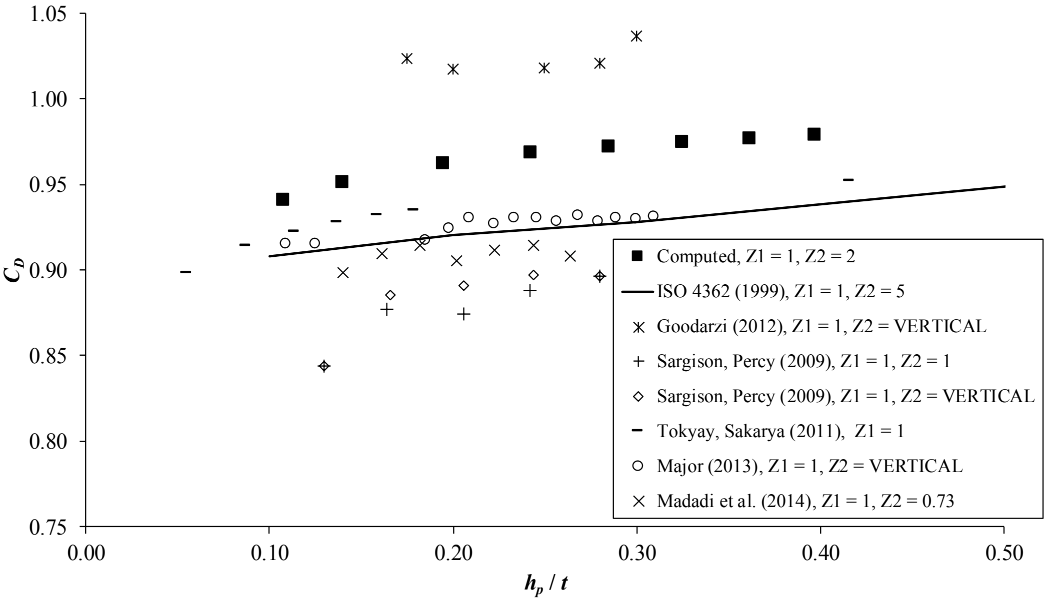

Fig. 4. Comparison between computed discharge coefficients and existing empirical results for upstream slope of $Z_1 = 1$.

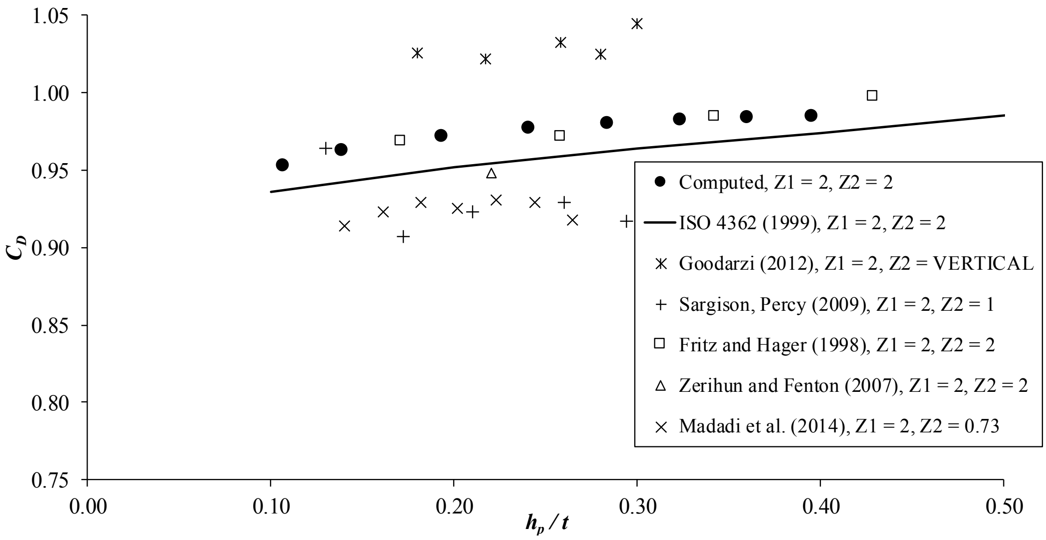

Fig. 5. Comparison between computed discharge coefficients and existing empirical results for upstream slope $Z_1 = 2$.

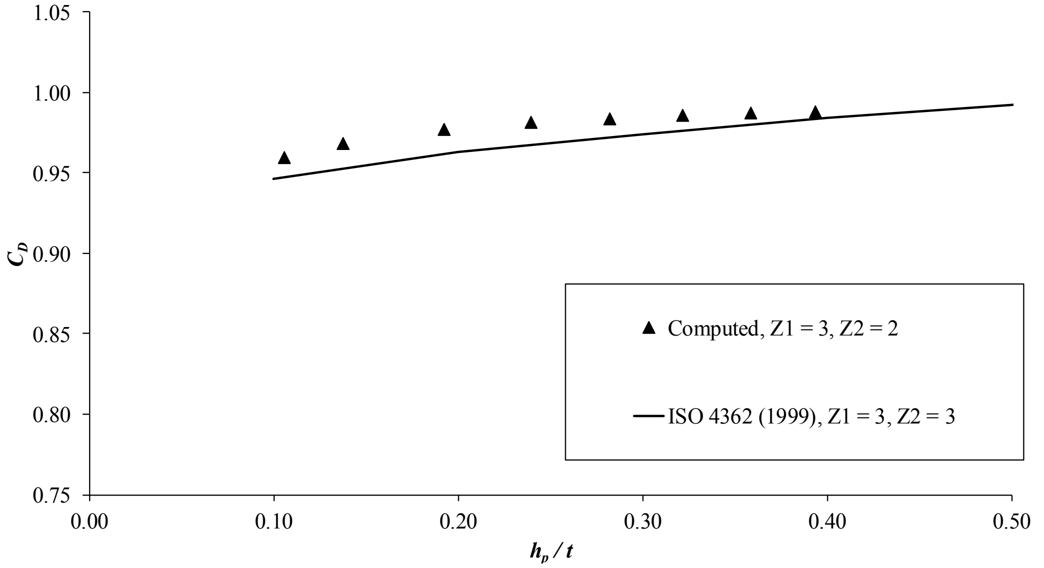

Fig. 6. Comparison between computed discharge coefficients and existing empirical results for upstream slope of $Z_1 = 3$.

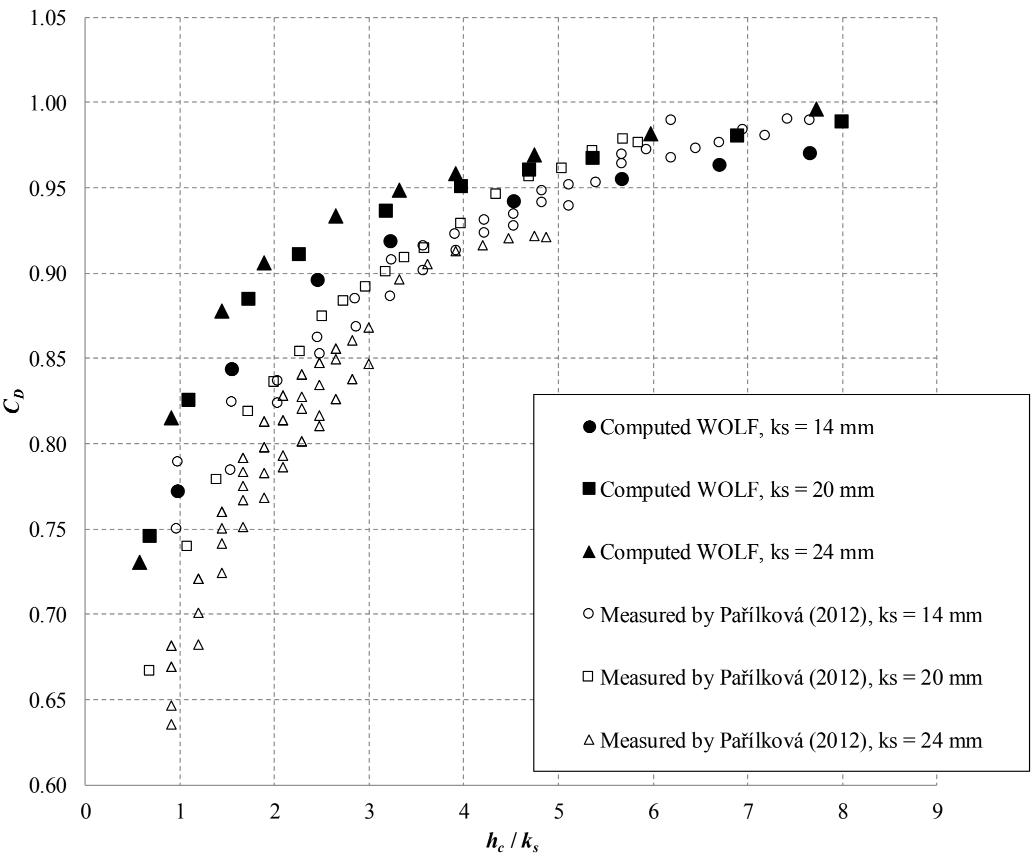

Fig. 7. The comparison of C_D obtained from shallow-water flow simulations (WOLF) and measured values.

CONCLUSIONS

The comparison of results of numerical computations with previously published experimental results was carried out for the evaluation of the discharge coefficient of frontal broad-crested weirs. The analysis was carried out for various weir geometries (upstream slopes), a range of specific discharge and considering both smooth and rough weir crest surface.

The discharge coefficients C_D obtained from numerical simulations were compared with values taken from literature (Fritz and Hager, 1998; Goodarzi et al., 2012; ISO 4362, 1999; Madadi et al., 2014; Major, 2013; Sargison and Percy, 2009; Tokyay and Altan-Sakarya, 2011; Zerihun and Fenton, 2007) and former own laboratory research (Pařílková et al., 2012). For the smooth weir surface, the flow simulations overestimate C_D by 3% to 6%, depending on upstream weir slope and the ratio h_p/t. Numerical simulations showed no effect of the downstream slope of the broad-crested weir, consistently with the experiments. For the rough weir crest, the overestimation of the discharge coefficient drops with an increasing ratio h_c/k_s.

The overestimation of the discharge coefficient in all simulated scenarios may be attributed to nonfulfillment of the assumption of uniform velocity and hydrostatic pressure along the vertical and to “missing” losses due to vertical contraction, which is not incorporated at the standard shallow flow equations (Zerihun and Fenton, 2007).

The results of the study provide practitioners with a quantification of the uncertainties arising when numerical shallow flow models are used at vertical contractions. In case of upstream weir slopes milder than 1:3, the error remains smaller than 2% and the shallow flow model may be used with an accuracy certainly suitable for a broad range of engineering applications. In contrast, in case of slopes steeper than 1:1, the error rapidly increases above 5% and modellers should be careful when simulating flow over steep obstructions, namely when h_p/t is close to 0.2.

Future research will focus on more complex configurations involving e.g. broad-crested side weirs.

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