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Vertical sorting in collisional layer of bimodal sediment transport

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Abstract: Intense collisional transport of bimodal sediment mixture in open-channel turbulent flow with water as carrying liquid is studied. The study focusses on steep inclined flows transporting solids of spherical shape and differing in either size or mass. A process of vertical sorting (segregation) of the two different solids fractions during the transport is analyzed and modelled. A segregation model is presented which is based on the kinetic theory of granular flows and builds on the Larcher-Jenkins segregation model for dry bimodal mixtures. Main modifications of the original model are the carrying medium (water instead of air) and a presence of a non-uniform distribution of sediment across the flow depth. Testing of the modified model reveals that the model is applicable to flow inclination slopes from 20 to 30 degrees approximately, making it appropriate for debris flow conditions. Changing the slope outside the specified range leads to numerical instability of the solution. A use of the bimodal mixture model is restricted to the grain size ratio 1.4 and no restriction is found for the grain mass ratio in a realistic range applicable to natural conditions. The model reveals trends in the vertical sorting under variable conditions showing that the sorting is more intense if flow is steeper and/or the difference in size or mass is bigger between the two sediment fractions in a bimodal mixture.

Keywords: Grain segregation; Bimodal mixture; Granular flow; Sheet flow; Sediment transport.

INTRODUCTION

Transport of sediment in environmental conditions has been subject to frequent investigations. Current climate developments with more frequent occurrences of extreme weather conditions leading to flash floods and landslides focus research attention to geomorphic flows with intense sediment transport. Typically for such flows, sediment grains of different properties are transported by water current in an open channel.

Grains of different sizes and/or densities are transported in flow of mixture composed of fluid and different fractions of grains. During the transport, grains of the different fractions can segregate forming regions a preferential presence of grains of the particular fractions across the flow depth. This phenomenon is observed in many natural processes (also some industrial processes) involving transport of solid grains. The segregation is affected by various conditions associated with the granular flow. If coarse solids fractions are transported in dense granular flows, then intergranular collisions have a major impact of the segregation. This type of segregation is known as kinetic sieving, intergranular percolation or gravity driven segregation (Frey et al., 2019) and a typical example of its occurrence is the dry gravity-driven flow recognized as snow- or rock avalanches on steep hillsides of mountains. One of the major forces, besides the gravitational force, acting on grains transported in avalanches or debris flows, is the force generated by intergranular collisions. The segregation itself affects properties of the flow and those mutual interactions must be considered in modelling of the segregation process in dense granular flows. Basically, there are two approaches to modelling of the segregation of colliding grains. One approach is the discrete element modelling (DEM) and it follows a movement of each individual grain in flow subjected to vertical sorting of grains, e.g. (van der Vaart et al., 2018; Zhao et al., 2019). The other approach considers a moving body of colliding grains as a continuum, while the flow is described using (time- and space-) averaged characteristics expressing the motion of individual grains.

Kinetic-theory (KT) based studies of solid-liquid flows dominated by intergranular collisions follow principles of the second of the two approaches and they have been conducted by several authors, Jenkins and Hanes (1998), Armanini et al. (2005), Berzi and Fraccarollo (2013) among others. A majority of the studies have been focused on collisions of mono-size grains. Just recently, more attention has started to be paid to collisional transport involving vertical sorting of grains of different properties. Naturally, it was methodologically correct that first studies focused on dry bimodal flows. Larcher and Jenkins (2013) proposed a segregation model for a prediction of the final stage of segregation in dry inclined flows of two types of spheres with same value of a restitution coefficient, following works of Silbert et al. (2001), Arnarson and Jenkins (2004), and others. The segregation model was further elaborated (Larcher and Jenkins, 2015) into two versions, again for dry inclined flows of binary mixtures of spheres, one predicting the time- (unsteady) evolution of the segregation process and the other describing the steady spatial (longitudinal) evolution of the segregation in the gravity-driven flow.

It is of practical importance to include the condition of a presence of water as carrying fluid in dense granular flow dominated by collisions (the condition is typical for instance for intense transport of bed load during flash floods on mountain streams). The presence of water complicates the modelling as the role of fluid viscosity and buoyancy must be considered. Recently, Larcher and Jenkins (2019) published a model to predict the final stage of segregation in flow of water and bimodal granular mixture.

In our previous works, we have been looking at conditions and mechanisms of intense bed load in steady turbulent open-channel flow of water carrying grains above the eroded plane bed. Results of our laboratory-experiment based investigations for mono-size model sediments (fractions of plastic grains) have been published in a number of papers, starting with (Matoušek et al., 2015). Our investigations have further extended to bimodal granular mixtures composed of fractions of the model lightweight sediments (Zrostlík and Matoušek, 2016). The fractions differed primarily in size (and color, which made observations of vertical sorting easy). The laboratory experiments with bimodal mixtures revealed a development of the interfacial layer between the granular bed and the collisional transport layer. This layer, in which grains slid over each other, was almost exclusively occupied by grains of the finer of the two fractions transported as bed load in the observed flow. We analyzed the flow at steady state after grain segregation was finished. The overall flow structure consisted of clear water as the upper layer, middle collision layers and lower deposit. The segregation of grains takes place only in the middle part.

Our modelling efforts in the field of dense gravity-driven aqueous bimodal mixtures started with works on a modification of the model by Larcher and Jenkins (2015) for the time evolution of segregation in a bimodal mixture. The modification introduces water as carrying liquid for the dense granular flow subjected to vertical sorting of grains (Zrostlík and Matoušek, 2017). The aim of this work is to present a complete modification of the Larcher-Jenkins model and to discuss the range of conditions to which it can be applied.

Before the modified model is introduced, it is useful to summarize principles and equations of the original model for dry bimodal flow.

SORTING MODEL FOR COLLISIONAL DRY FLOW OF BIMODAL MIXTURE

The Larcher-Jenkins model (2015) considers dry flow of a mixture of two size spherical grains A and B with radii r_A and r_B, mass densities ρ_A and ρ_B, masses m_A and m_B, and number densities $n_A = \rho_A / m_A$ and $n_B = \rho_B / m_B$. The local density number of mixture $n = n_A + n_B$ at each position within the transport layer. The density number is related to the local volumetric concentration of the A-fraction through $c_A = 4\pi n_A r_A^3 / 3$. A parameter called the measure of segregation X is defined as $X \equiv (n_A - n_B)/2n$ in the model.

Model equations

The equations below are from Larcher and Jenkins (2015). For uniform time-dependent segregation in bimodal mixture, the following mass balance equation solves sorting in the mixture with respect to time t and position above the flow bottom y,

$$\rho \frac{\partial X}{\partial t} + \frac{\partial}{\partial y}\left[\frac{m_A n}{4}\left(1-4X^2\right)\left(v_A - v_B\right)\right] = 0 \tag{1}$$

In Eq. (1), the y-axis is normal to the main flow direction. This balance equation in its entirety considers sorting in time and also in space $\rho = \rho_A + \rho_B$.

The following relations are based on the kinetic theory and express quantities D_{AB} and $(v_A – v_B)$ for the condition of dry collisions. A relation for the difference in the vector diffusion velocities $(v_A – v_B)$ was derived originally by Arnarson and Jenkins (2004) and modified by Larcher and Jenkins to the form

$$(v_A - v_B) = -D_{AB}\left[\begin{array}{l} \left(\Gamma_1 \delta m + R_1 \delta r\right)\dfrac{\nabla T}{T} - \\ \left(\Gamma_2 \delta m + R_2 \delta r\right)\dfrac{m_{AB}\cos\phi}{2T} + \\ \dfrac{\nabla X}{0.25 - X^2} \end{array}\right]. \tag{2}$$

The measure of size difference δr is determined from $\delta r \equiv \left(r_A / r_B\right) - 1$ and the measure of mass ratio $\delta m \equiv \left(m_A - m_B\right)/m_{AB}$. The quantity m_{AB} is the sum of mass of the two fractions and r_{AB} is the sum of radii. The diffusivity coefficient D_{AB},

$$D_{AB} = \frac{\pi^{1/2}}{16}\frac{r_{AB}}{G}\left(\frac{2T}{m_{AB}}\right)^{1/2}, \tag{3}$$

and additional coefficients for Eq. (2) also originate in the kinetic theory,

$$\Gamma_1 = \frac{179}{29}G + \frac{105}{116} \doteq 6.17G, \tag{4}$$

$$\begin{aligned} R_1 = {} & \frac{5}{58}\left[2 + \frac{c(3-c)}{2-c} - \frac{12}{5}G\right] + \\ & 2G\left[3 + \frac{c(3-c)}{2-c}\right] - \frac{12cH(1+4G)}{1+4G+4cH} \doteq -4.35G, \end{aligned} \tag{5}$$

$$\Gamma_2 = 2, \quad R_2 = -\frac{12cH}{1+4G+4cH} \doteq -3, \tag{6a, b}$$

$$c = \frac{c_M G}{G + 5.69\left(c_M - 0.49\right)}. \tag{7}$$

The individual coefficients are simplified as in the original model. The simplification is justified as its effect on values of the coefficients is small. The granular temperature T for Eqs. (2) and (3) is expressed using the following relation which considers a mixture with a uniform concentration distribution across the flow depth (Silbert et al., 2001) and it is derived by a mathematical simulation of 3D motions and collisions of grains,

$$T = \frac{m_{AB}(h-y)}{4(1+e)G} g\cos\phi\left(1 + 2X\delta m\right) \tag{8}$$

For Eq. (8), the parameter G is determined by an expression based on the same mathematical simulation,

$$G = \left\{\frac{4J}{5\pi^{1/2}}\frac{1}{1+e}\left[\frac{15}{J}\frac{\left(1-e^2\right)}{\alpha}\right]^{1/3}\frac{1}{\tan\phi}\right\}^9 \left[1 + 3X\left(\delta r + \delta m\right)\right] \tag{9}$$

in which e is the coefficient of restitution and ϕ is the angle of inclination. The function H is related to radia distribution function of the mixture and it can be expressed as $H = \partial G / \partial c$. The coefficient J for very dissipative spheres reads

$$J = \frac{(1+e)}{2} + \frac{\pi}{4}\frac{(3e-1)(1+e)^2}{\left[24 - (1-e)(11-e)\right]} \tag{10}$$

From Eqs. (1) to (10), the final equation for the vertical sorting reads

$$\frac{\partial X}{\partial t}=\frac{r_{AB}(\pi g\cos\phi)^{1/2}}{128G^{2/3}}\left(\frac{2}{1+e}\right)^{1/2}\frac{\partial}{\partial y}\left[(h-y)^{1/2}\left\{\left[\left(2(1+e)G\Gamma_2-\Gamma_1\right)\delta m+\left(2(1+e)GR_2-R_1\right)\delta r\right]\frac{1-4X^2}{h-y}+4\frac{\partial X}{\partial y}\right\}\right] \quad (11)$$

The sorting formula (Eq. (11)) is converted to the dimensionless form by normalizing the depth and time using $z\equiv y/h$, $\tau\equiv t/(r_{AB}/g)^{1/2}$. The sorting parameter X transforms to ζ which ensures a conservation of the total number of grains over the height of flow, $\frac{2cX}{(\hat{c}_A+\hat{c}_B)}=\zeta$ in which $\hat{c}_A\equiv\frac{\bar{n}}{\bar{n}_A}\bar{c}_A$. Hence, the final relation for vertical sorting is reached in the form of the parabolic-elliptic partial differential equation,

$$\frac{\delta\zeta}{\delta\tau}=\left(\frac{r_A+r_B}{h}\right)^{3/2}\frac{(\pi\cos\phi)^{1/2}}{128G^{3/2}}\left(\frac{2}{1+e}\right)^{1/2}\frac{2c}{(\hat{c}_A+\hat{c}_B)}\times$$
$$\times\frac{\delta}{\delta z}\left\{\frac{\left[(2(1+e)G\Gamma_2-\Gamma_1)\delta m+(2(1+e)GR_2-R_1)\delta r\right]}{(1-z)^{1/2}}\right. \quad (12)$$
$$\left.\times\left[1-\frac{(\hat{c}_A-\hat{c}_B)}{c^2}\zeta^2\right]+2(1-z)^{1/2}\frac{(\hat{c}_A-\hat{c}_B)}{c}\frac{\delta\zeta}{\delta z}\right\}.$$

A prediction of vertical sorting by the Larcher-Jenkins model requires to find a solution for Eq. (12).

Modelled conditions

The original conditions considered by Larcher and Jankins model of dry flow are:

- Dry gravity-driven flow on inclined plane,
- grains of spherical shape,
- mixture of two fractions with not much different size and mass (the max relative size 1.3 was tested by Larcher and Jenkins 2015),
- uniform distribution of grains across flow depth, i.e. constant concentration profile (see Figure 1),
- velocity profile using Eq. (12) from Larcher and Jenkins (2013), see Figure 1,

$$u=u_0+\frac{5\pi^{1/2}}{6J}\frac{1}{r_{AB}}\left(\frac{1+e}{2G}g\cos\phi\right)^{1/2} \quad (13)$$
$$\left[h^{3/2}-(h-y)^{3/2}\right]\tan\phi(1-X\delta r)$$

Figure 1 shows the model assumptions for vertical segregation conditions in bimodal mixture.

MODEL MODIFICATION FOR WET SORTING (WATER AS CARRIER)

In aqueous flow transporting in bed load, a concentration profile can be considered as approximately linear across the collisional transport layer through which grains are transported in the upper plane bed regime, e.g. (Capart and Fraccarollo, 2011; Matoušek et al., 2015). For modelling of the segregation process in bimodal bed load, we focus on the collisional transport layer in which the segregation takes place.

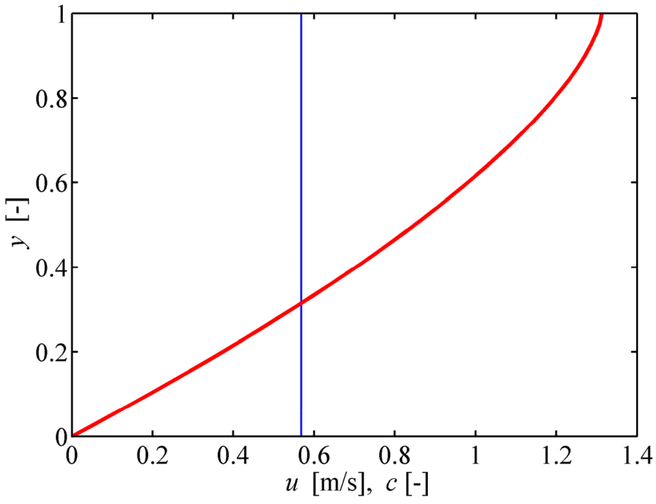


Fig. 1. Velocity profile (using Eq. (13)) and total concentration profile for e = 0.65, c_M = 0.586, r_B = 1.1r_A, ϕ = 25°. Legend: red line - velocity profile, blue line – total concentration profile for mixture of two fractions.

Modifications in model equations

We assume a linear concentration profile with zero at the top of the collisional layer and the maximum local concentration at the bottom of the collisional layer. It is determined by Eq. (7).

A modification is required in Eq. (12) to accommodate the presence of the linear concentration profile (the local concentration varies with the vertical position). In principle, the variable local concentration affects other c-related parameters, which become sensitive to a vertical position in the flow. Unfortunately, modifications for the other parameters made the model computationally unstable and had to be abandoned. Therefore, the height variation was introduced only for the local concentration c. The other c-related parameters of the segregation model (Γ_1, Γ_2, R_1, R_2, G) use a value of the flow-depth averaged concentration at the initial condition instead of c.

Further mathematical complications were associated with values of local concentration at the boundaries of the collisional layer, leading to dividing by zero in the model equations. Therefore, computations were carried out only in the range of the dimensionless vertical positions between 0.1 and 0.9. The boundary conditions were set so that the granular fluxes were zero at the top and at the bottom of the collisional layer. The initial conditions for each calculation included the constant ratio of volumes of both fractions at every height position and the perfect mixing of fractions at the initiation of the segregation process. The following paragraph and Figure 2 summarize the conditions.

Modelled conditions

Conditions of use for modified model according to water suspension with bimodal mixture of particles:

- wet gravity-driven flow on inclined plane,
- fully developed turbulent flow,
- grains of spherical shape,
- mixture of two fractions with not much different size and mass (quantification will follow),
- linear distribution of grains across transport layer (see Figure 2),
- velocity profile using Eq. (13), see Figure 2.

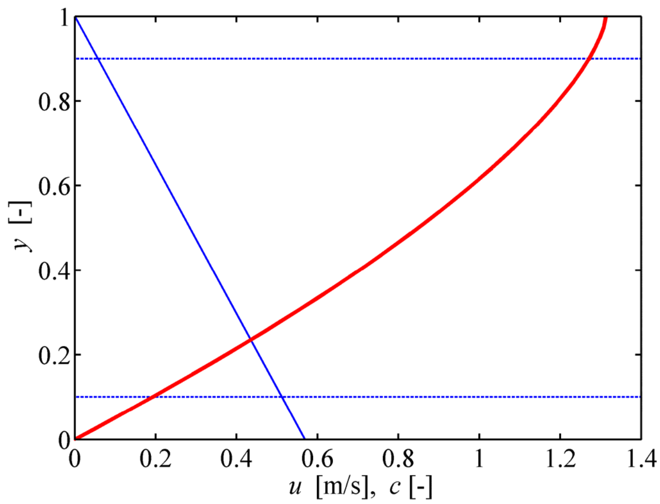


Fig. 2. Initial profiles of velocity and total concentration. Legend: red line – velocity profile, inclined blue line – concentration profile, horizontal blue lines – range of computation.

RESULTS OF MODIFIED MODEL AND THEIR DISCUSSION

The model solves Eq. (12) in time steps using the Matlab solver called pdepe. The solver is constructed for boundary value problem for initial boundary condition. The boundary value of ξ is 1 at the top of normalize transport layer, i.e. at $z = 1$. At the bottom of flow ($z = 0$), $\xi = -1$.

A model simulation produces a prediction of a distribution of the sorting parameter X, representing the effort of sorting of the individual grains. In order to interpret a simulation result in the form of concentration profiles of the individual fractions, the following equations are used,

$$c_A = -\frac{(2X+1)c}{(2X+1)-(2X-1)(r_B/r_A)^3} \quad (14a)$$

$$c_B = \frac{(2X-1)c}{(2X-1)-(2X+1)(r_A/r_B)^3} \quad (14b)$$

Profiles of these concentrations are presented for the final state of vertical sorting in figures below. Also plotted as model results are graphs showing the speed of sorting, i.e. time required to reach the final state of sorting. Effects of different grain size ratio, different mass ratio and slope of flow are discussed as well. Furthermore, possible ranges of values of individual parameters are determined applicable in the model solved with the mathematical solver pdepe.

Effect of flow slope on vertical sorting

An influence of the flow longitudinal slope, ϕ, on the granular sorting was tested first. In various simulations with variable ϕ, the grain size and mass were kept constant to isolate the slope effect. The range of slopes to which the model can be applied is found to be limited by values of 20 and 30 degrees. It turned out that there were two conditions restricting the use of the model for smaller slopes. The first restriction was caused by a structure of the computational solver, resulting in values sweeping in the area with sudden change of values. A possible solution for this restriction could be to use a computing network with more grid points. The second restriction is more significant. Sorting at smaller slopes is much slower and it is necessary to choose a much finer step in time discretization, which leads to a requirement of too high computational power for a common use.

Figure 3 plots final concentration profiles for fractions A and B in dimensionless time 6000 at three different slopes, 20, 26 and 30 degrees. The simulations show that a steeper slope produces a more intense sorting, even in the lower part of the collision layer. At the same time, a shallower slope creates a sharper interface between the sorted fractions.

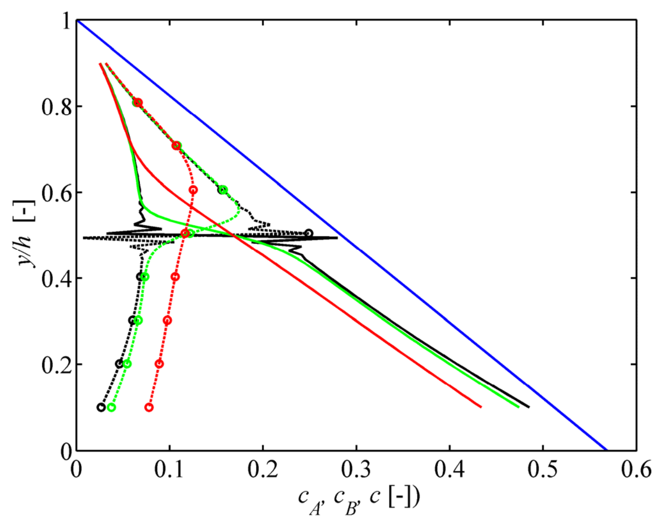


Fig. 3. Final concentration profiles of c_A and c_B, for $r_A = r_B$, $m_A = m_B$, at dimensionless time 6000 for 3 different flow slopes. Legend: inclined blue line – total concentration, lines with symbols – spheres A, lines without symbols - spheres B (black - $\phi = 20°$, green - $\phi = 26°$, red - $\phi = 30°$).

Figure 4 shows the time evolution of sorting for the three cases plotted in Figure 3. Each curve in Figure 4 represents the time development in the centre of gravity of a concentration profile of an individual fraction. Apparently, the speed of sorting increases with the flow slope.

Sorting by grain size

An effect of the relative size of grains of fractions A and B, r_A/r_B (i.e. the ratio of grain sizes of the two fractions), on the vertical sorting was tested by setting values of the relative size as the only variable in the model simulations. The results presented below are for the slope $\phi = 25$ degree. The range of relative sizes was set from 1 to 1.4. It made no sense to test values smaller than 1 (they do not exist) and the sizes bigger than 1.4 resulted in a reverse sorting tendency at the upper border of the simulated layer. The value of 1.4 also corresponded with suggestions for the relative size in the original model. Furthermore, no experimental data are available to verify sorting for high relative size mixtures.

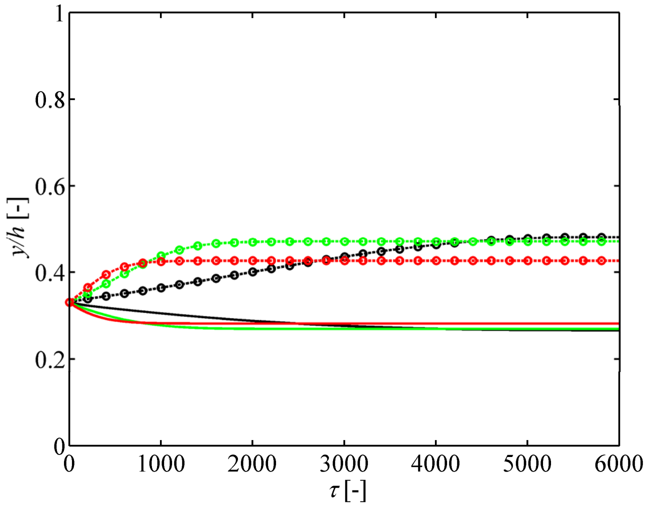


Fig. 4. Time evolution of gravity centroids of concentration profile of c_A and c_B, for $r_A = r_B$, $m_A = m_B$, for 3 different flow slopes as in Figure 3. Legend: increasing curves with symbols = fraction B, decreasing curves = fraction A (black - ϕ = 20°, green - ϕ = 26°, red - ϕ = 30°).

Figure 5 shows the effect of the relative grain size on the vertical sorting in the form of the final concentration profiles for both fractions and of the time evolution of the center of gravity of the profiles. A special case is the condition of the relative size being unity. This exhibits the evolution of sorting only on the basis of the flow itself. If the relative size increases (to 1.1 and 1.4), then the sorting becomes more intense (the right-hand side panel of Figure 5). At the same time, the relative size has a limited effect on the speed of sorting as the right-hand side panel of Figure 5 shows.

Sorting by grain mass

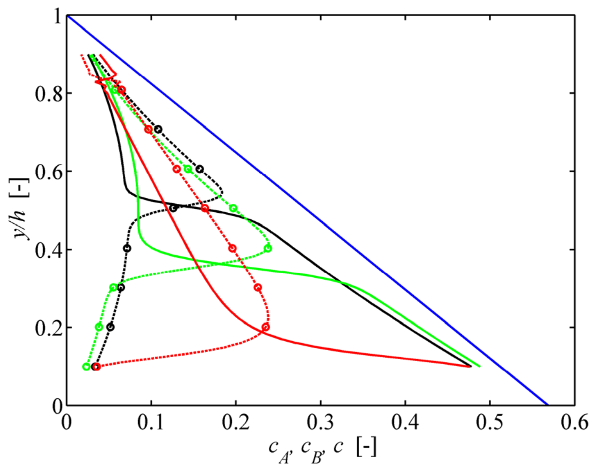
The last tested parameter was the mass ratio, m_A/m_B. Its effect on the vertical sorting was tested by setting values of the mass ratio as the only variable in the model simulations. The results presented below are for the slope ϕ = 25 degree and the grain size ratio r_A/r_B = 1. The range of mass ratios was set from 1 to 5, which was considered to cover the range expected in engineering applications. Figure 6 demonstrates that the larger m_A/m_B the stronger the vertical sorting throughout the collisional transport layer. The effect of m_A/m_B on the time evolution of the separation is relatively weak although considerably stronger than the effect of the size ratio.

CONCLUSIONS

The Larcher-Jenkins segregation model for dry bimodal granular flow is modified to predict vertical sorting in collisional transport layer of bimodal mixture flow with water as a carrying fluid and for a condition of a linear distribution of grains instead of the uniform distribution considered in the original model. The modifications were motivated by experimental observations in our laboratory flume. A testing of the ranges of applicability of the wet sorting model showed that the modified model could be used in the range of slopes from 20 to 30 degree, making it suitable for debris flow conditions. An extension of the model applicability to flatter slopes was restricted by the applied mathematical solution using the Matlab solver called pdepe. A further restriction is the relative size of two granular fractions in bimodal mixture, the testing confirmed that the size ratio should not be bigger than 1.4. No restriction was found for the mass ratio as the model worked also for the maximum set value of 5, which covers all conditions of real-world applications.

The model results suggested that the larger the ratios of mass and size the stronger is the vertical sorting in the lower part of the collisional transport layer in the aqueous mixture flow.

Furthermore, a numerical testing of time evolution of concentration profiles showed that the longitudinal slope of the flow had a considerably greater effect on the vertical sorting than the grain size ratio and the grain mass ratio of the particular fractions.



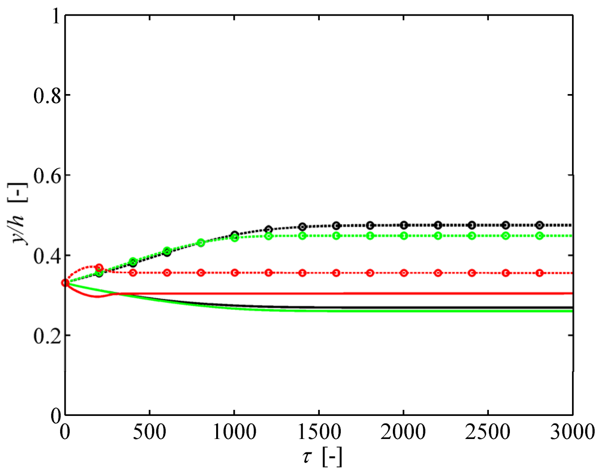


Fig. 5. Left panel: final concentration profiles for different relative grain sizes r_A/r_B, for ϕ = 25°, $m_A = m_B$. Legend: lines –spheres A, lines with symbols - spheres B (black – r_A/r_B = 1, green - r_A/r_B = 1.1, red - r_A/r_B = 1.4).
Right panel: time evolution of gravity centre of concentration profile, for ϕ = 25°, $m_A = m_B$. Legend: increasing curves with symbols = fraction B, decreasing curve = fraction A (black - r_A/r_B = 1, green - r_A/r_B = 1.1, red - r_A/r_B = 1.4).

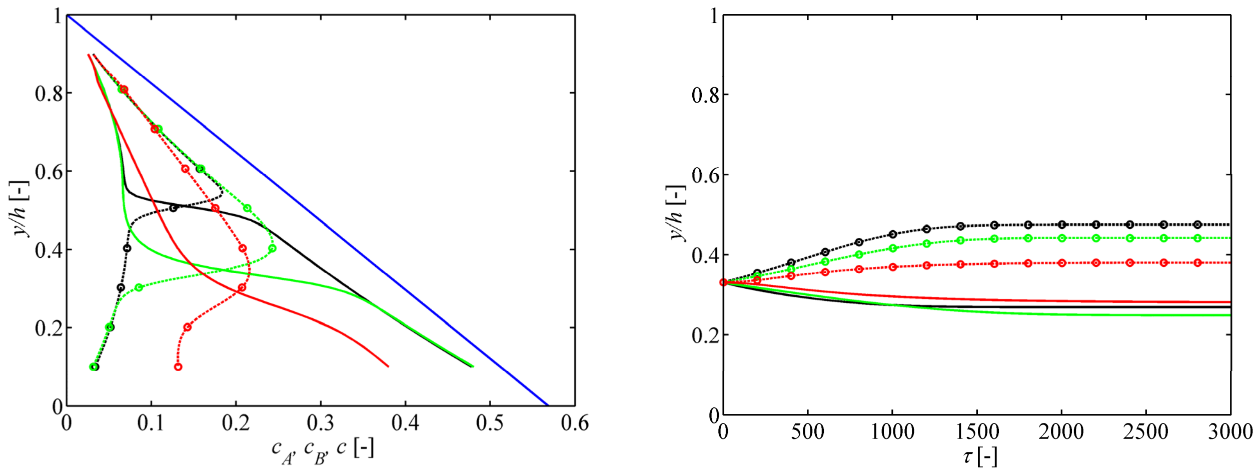


Fig. 6. Left panel: final concentration profile for different mass ratios m_A/m_B, for $\phi = 25°$, $r_A/r_B = 1$. Legend: lines with symbols –spheres A, lines - spheres B (black – $m_A/r_B = 1$, green $m_A/m_B = 1.5$, red - $m_A/m_B = 5$).
Right panel: time evolution of gravity centre of concentration profiles, for $\phi = 25°$, $r_A/r_B, = 1$. Legend: increasing curves with symbols = fraction A, decreasing curves = fraction B (black - $m_A/m_B = 1$ green line - $m_A/m_B = 1.5$, red - $m_A/m_B = 5$).

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NOMENCLATURE

Remark: symbols with over-line are depth-averaged variables and symbol with roof are variables normalized by depth-averaged variables

A, B – index for fractions of spheres
c [–] – total volumetric concentration
c_M [–] – maximum mixture concentration
D_{AB} [–] – diffusivity coefficient
e [–] – coefficient of restitution
G [–] – concentration-related function
g [m/s^2] – gravitational acceleration
H [–] – concentration-related function
h [m] – depth of flow
J [–] – coefficient for mixture shear stress
m [kg] – mass of grain
m_{AB} [m] – count of mass of grains A and B
n [m^{-3}] – number density
$R_{1,2}$ [–] – function of mixture concentration
r [m] – radius of grain
r_{AB} [m] – count of radii of grains A and B

T [m^2/s^2] – granular temperature
t [s] – time
u [m/s] – local velocity in longitudinal direction of flow
u_0 [m/s] – slip velocity
X [–] – measure of segregation
y [m] – vertical position
z [–] – dimensionless vertical position
α [–] – dimensionless coefficient of order unity
Γ [–] – rate of collisional dissipation as function of concentration
δr [–] – measure of size difference
δm [–]– measure of mass difference
ϕ [°]– flow slope (angle of longitudinal inclination)
τ [–] – dimensionless time
ρ [kg/m^3] – density of grain
v [m/s] – vertical diffusivity velocity
ζ [–] – dimensionless measure of segregation

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