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Validation of friction factor predictions in vertical slurry flows with coarse particles

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Abstract: The paper presents validation of a mathematical model describing the friction factor by comparing the predicted and measured results in a broad range of solid concentrations and mean particle diameters. Three different types of solids, surrounded by water as a carrier liquid, namely Canasphere, PVC, and Sand were used with solids density from 1045 to 2650 kg/m^3, and in the range of solid concentrations by volume from 0.10 to 0.45. All solid particles were narrowly sized with mean particle diameters between 1.5 and 3.4 mm. It is presented that the model predicts the friction factor fairly well. The paper demonstrates that solid particle diameter plays a crucial role for the friction factor in a vertical slurry flow with coarse solid particles. The mathematical model is discussed in reference to damping of turbulence in such flows. As the friction factor is below the friction for water it is concluded that it is possible that the effect of damping of turbulence is included in the K_B function, which depends on the Reynolds number.

Keywords: Flow with coarse particles; Particle-wall stress; Modelling of vertical flow.

INTRODUCTION

Solid-liquid flow in a pipeline exists widely in the chemical and mining industry and it is still an ecological alternative when compared to traditional forms of transportation. It is well known that the size of solid particles strongly affects frictional losses in a pipeline, therefore, the first step should be to identify the particles' size and their concentration (Shook and Roco, 1991; Sumner at al., 1990; Sumner, 1992).

The transport of coarse particles, using liquid as a carrier phase, requires careful consideration and the analysis of numerous factors, such as the constitutive relation between stress and deformation, the particles' diameter, solid concentration, deposition velocity, solid and liquid properties, as well as *particle – fluid*, *particle – particle*, and *particle – wall* interactions, and also adequately matched characteristics of the pipeline and the pump (Peker and Helvaci, 2008; Wennberg, 2010; Wilson et al., 2006).

Slurry flow mainly occurs in horizontal pipelines, however, vertical hydraulic transport over long distances is of special interest in the fields of dredging and ocean mining (Van Wijk, 2016; Van Wijk et al., 2014). One of the earlier researchers dealing with solid–liquid flow in vertical pipes was Wing (1972). The author considered hydraulic transport of marine minerals from the seabed to the sea surface. The test parameters that varied were solids with particle diameters of 0.325 mm and 0.749 mm, pipes with diameters of 0.023 m and 0.048 m, bulk velocities up to 0.305 m/s, and solid concentrations by volume up to 0.30. The researcher concluded that within the parameter limits of this experiment there is no evidence to indicate that the solids being transported contribute any extra friction loss to the normal plain water friction loss associated with pipe flow.

If turbulent flow with sufficiently small solid particles is considered, i.e. particles can move freely inside the viscous sublayer, the friction process proceeds similarly to a single-phase flow (Coulson et al., 1996). When solid particles are larger than the thickness of the viscous sublayer, like coarse–dispersion slurry, their contact with the pipe wall is limited due to the emerging *lift forces* that push solid particles from the wall. Therefore, assuming there is limited contact of the solid phase with the pipe wall, it can be presumed that the wall shear stress should be similar to the flow of the carrier liquid. We know examples in the literature that in some cases the frictional head loss, in turbulent slurry flow in a vertical pipe, is similar or below that for carrier liquid. This was proven by experiments in a vertical slurry flow of sand–water mixture, conducted by Charles and Charles (1971) for d_{50} = 0.216 mm, Ghosh and Shook (1990) for d_{50} = 0.6 mm, Sumner (1992) for d_{50} = 0.47, Matousek (2005) for d_{50} = 0.37 mm, Talmon (2013) for d_{50} = 0.1–2 mm. The problem of defining the boundary above which the influence of the *solid phase – solid phase* interaction is dominant in the flow was provided by Caulet et al. (1996). Their research shows that when the solid concentration is greater than 0.20 by volume, the dominant stresses in the flow are *solid phase – solid phase*.

Shook and Bartosik conducted experimental studies of turbulent flow of slurry in a vertical pipe, with solid particles of density similar to water (ρ_P = 1045 kg/m^3), with mean solid particles diameter of d_{50} = 1.5 mm, showing that in the range of solids concentration $0 < C_V \leq 0.30$, the frictional head loss is lower than for a single-phase flow of the carrier liquid (Shook and Bartosik, 1994). Thus, it follows that a damping effect of the contact of solid particles with the pipe wall could take place. Similar experimental studies carried out for particles of identical density, but with a diameter d_{50} = 2.8 mm, showed that the frictional head losses are lower than for water only for $C_V \leq 0.10$ (Shook and Bartosik, 1994). However, for $C_V > 0.10$, the frictional head losses increased linearly and then exponentially as the solid's concentration increases. They concluded that:

– the wall friction in a turbulent vertical flow is strongly influenced by the diameter of the particles,

in the case of flow with a sufficiently large particles diameter and particles density (d_{50} > 1.5 mm; S_p >> 1) and sufficiently high solid concentrations ($C_V \geq 0.20$), the importance of the transport mechanism is played by the *solid particle – pipe wall* interaction, which causes an increase of wall shear stress when compared to carrier liquid flow.

Of course, the flow mechanism is much more complex as we know that solid particles can increase or attenuate the turbulence. Schreck and Kleis (1993) studied the movement of solid particles with a diameter of 0.65 mm, which were polystyrene and glass with a density ratio of S_ρ = 1.045 and 2.40, respectively. They used a Doppler Laser Anemometer. Their analysis showed that the solid particles follow liquid only partially, therefore, the fluctuating velocity components are smaller in relation to the carrier liquid, even when the density of the solid particles is similar to the density of the liquid. Faraj and Wang (2012) conducted laboratory experiments carried out on an open flow loop using Electrical Resistance Tomography (ERT) to interrogate the internal structure of horizontal and vertical slurry flow. They used the Fast Impedance Camera System, with a temporal resolution up to 1000 dual–frames per second. A set of experiments was carried out on coarse and medium particles of *sand–water* slurry flows with 2% and 10% throughput volumetric concentration and the transport velocity in the range of 1.5 – 5 m/s. The authors found that the coarse sand slurry flows in a plug or core flow pattern, whereas the flow of medium sand slurry demonstrated an annular-like flow pattern. Van Dijke (2010) conducted similar studies and developed a one-dimensional steady state model of the transport of differently sized solid particles in a vertical riser, which showed that concentration peaks could develop during transport. Concluding, one can say that phenomenon of slurry flow with coarse particles is very complex and still difficult for mathematical modelling of the frictional head loss. One crucial barrier for building reliable mathematical models is limited access to experimental data, especially regarding viscous and buffer layers with solid concentrations above 0.2 by volume.

If coarse-dispersed slurry flow is considered, we know that it is impossible to measure slurry viscosity as the sedimentation process is substantial. For this reason, predictions of frictional head loss assume that slurry viscosity is equal to the viscosity of the carrier liquid and the density is equal to the slurry density. For such cases, it is usually assumed that the pressure drop of a slurry with coarse particles can be determined as:

$$\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_m = \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_L \frac{\rho_m}{\rho_L} \tag{1}$$

From Equation (1) it follows that the density of the slurry is the main factor determining pressure losses. The difference between the pressure drop for the slurry $(\mathrm{d}p/\mathrm{d}x)_m$ and the carrier liquid $(\mathrm{d}p/\mathrm{d}x)_L$ is called the *solid phase effect*. However, with the increase of the particle diameter and/or the solid concentrations, this approach shows discrepancy between measurements and predictions.

The availability of mathematical models, which predict a vertical flow of slurry with coarse particles is very limited. For example, Krampa–Morlu et al. (2004) used CFX 4.4 (ANSYS Inc.) software to predict the velocity profiles of a turbulent slurry flow with coarse particles in an upward vertical pipeline. Two sets of sand particle diameters were used in this test: d_{50} = 0.47 mm and d_{50} = 1.7 mm, with a density of 2650 kg/m^3. The simulations were carried out for solid concentrations $0 < C_V < 0.30$. The results of numerical calculations were compared with the results of the experimental data of Sumner et al. (1990). The authors admitted that the simulation results significantly differ from the results of measurements.

The mathematical modelling of solid–liquid flow with coarse particles is still a long way from the knowledge gathered for single–phase flows (Bartosik and Shook, 1995; Messa and Malavasi, 2013; Miedema, 2015). The mathematical models, which are available in the literature, are usually not validated for industrial applications, which include a broad range of solid concentrations. It has been the endeavour of researchers around the world to develop accurate models to predict pressure drop and velocity distribution in a solid–liquid flow in pipelines. The friction factor is one of the most important technical parameters to be evaluated by the designers for designing a pipeline transportation system in a deep ocean, and the parameter which dictates frictional head loss and selection of the pump capacity (Rabinovich et al., 2012; Talmon and Rhee, 2011; Wilson et al., 2006). Therefore, the paper presents a predicted friction factor in an upward vertical pipeline for slurry flow with coarse solid particles and compares the results of predictions with measurements. Predictions of the friction factor were made using the mathematical model developed by Shook and Bartosik (1994) and next improved by Bartosik (1996). The mathematical model uses the Bagnold concept (Bagnold, 1954) and was detailly described by Bartosik (2009).

The paper presents a validation of the mathematical model describing the friction factor by comparing the predicted and measured results in a broad range of solid concentrations and mean solid particle diameters. Three different types of solids, surrounded by water as a carrier liquid, namely Canasphere, PVC, and Sand were used with solids density from 1045 to 2650 kg/m^3, and in the range of solid concentrations by volume from 0.10 to 0.45. It is demonstrated that the predicted friction factor matching well measurements, however, some exceptions exist. The paper demonstrates that mean solid particle diameter plays a crucial role in a vertical slurry flow with coarse particles.

The aim of the paper is to validate the mathematical model, which predicts the friction factor, by comparing predictions with measurements in vertical pipeline for up-ward slurry flow with coarse particles if additional stresses due to *particle – wall* interactions are included. It must be noted that such a comparison is not available in the literature.

The mathematical model

If vertical pipe flow of medium and coarse solid particles is considered it should be noted that several models are available in literature, as for instance:

- Shook and Bartosik (1994) and Bartosik (1996) for d_{50} = 1.4 – 3.4 mm
- Ferre and Shook (1998) for d_{50} = 1.8 – 4.6 mm
- Gillies and Shook (2000) for d_{50} = 0.175 mm
- Matousek (2005) for d_{50} = 0.37 mm
- Talmon for d_{50} = 0.1 – 2 mm

For the purpose of this research, the friction factor for slurry with coarse particles will be calculated using the mathematical model developed by Shook and Bartosik (1994) and next improved by Bartosik (1996), which compact delivery is presented below.

Let's consider up-ward slurry flow in a vertical straight pipeline of constant diameter. Assuming that the slurry flow is axially symmetrical (V = 0) and without circumferential eddies (W = 0), the linear momentum equation for quantities averaged over a pipe cross section can be expressed for main flow direction (ox), as follows (Longwell, 1966):

$$\rho\left(\frac{\partial U_b}{\partial t} + U_b\frac{\partial U_b}{\partial x} + g\frac{\partial h}{\partial x}\right) + \frac{\partial p}{\partial x} + 4\frac{\tau_w}{D} = 0 \tag{2}$$

Bulk velocity U_b is computed by integrating the local velocity

U across a pipe, as follows:

$$U_b = \frac{1}{A}\iint U\, dA \tag{3}$$

where A is cross section of a pipe and is constant (D = const).

Assuming that flow is stationary ($\partial/\partial t = 0$) and fully developed ($\partial U_b/\partial x = 0$), Equation (2) can be simplified, as follows:

$$\rho g \frac{\Delta h}{\Delta x} + \frac{\Delta p}{\Delta x} + \frac{4}{D}\tau_w = 0 \tag{4}$$

The first term in Equation (4) is called the gravitational term and is denoted as:

$$\rho g \frac{\Delta h}{\Delta x} = \frac{\Delta p^*}{\Delta x} \tag{5}$$

The gravitational term is equal to zero for horizontal flow and its importance increases with increase of pipe inclination. Taking into account Equation (5), the final form of Equation (4) is, as follows:

$$\frac{p_1 - p_2}{\Delta x} = \frac{\Delta p^*}{\Delta x} + \frac{4}{D}\tau_w \tag{6}$$

where $p_1 - p_2$ is the total static pressure drop in a vertical up-ward pipe flow – see Fig. 1.

Looking for possibility of comparing a vertical up-ward flow with a horizontal flow the gravitational term in Equation (6) will be subtracted. Equation (6) can be rewritten in the following form:

$$\frac{p_1 - p_2}{\Delta x} - \frac{\Delta p^*}{\Delta x} = \frac{4}{D}\tau_w \tag{7}$$

Left hand side of Equation (7) represents pressure drop for a horizontal pipe flow. Now we consider a horizontal flow, with data obtained from a vertical flow. Of course, we know that the vertical flow is axially symmetrical, even for the ratio of slurry to liquid density much higher than unity, while the horizontal flow is pseudo-homogenous or heterogeneous. Nevertheless, such treatment is valuable as we consider horizontal flow, which is fully axially symmetrical, so analysis of such flow is simpler. In a vertical flow we measured the total pressure drop ($p_1 - p_2$). Therefore, it is necessary to calculate the gravitational term in order to subtract him form the total pressure drop. To do that let's consider Fig. 1, which shows the method of calculation of the gravitational term in a vertical up-ward slurry flow. Let's choose cross sections 1–1 and 2–2 and a datum level, denoted as 0–0 (dashed line), as it is presented in Fig. 1. Assuming that we consider stationary state, it means that slurry velocity is zero, it is possible to develop the equilibrium equation, which is following:

$$p_2 + \rho_m g \Delta x + \rho_L g h = p_2 + \rho_L g\left(\Delta x + h\right) + \Delta p^* \tag{8}$$

where $\Delta x = H\text{–}h$ in accordance with Fig. 1.

From Equation (8) we can get final form of the gravitational term, which is, as follows:

$$\frac{\Delta p^*}{\Delta x} = g\left(\rho_m - \rho_L\right) \tag{9}$$

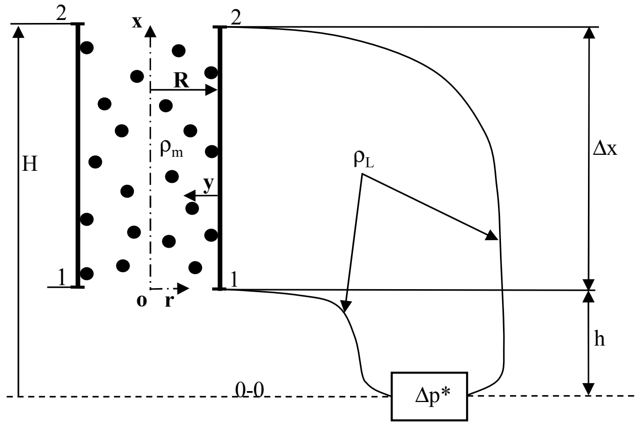

Fig. 1. Method of calculation of the gravitational term in a vertical up-ward slurry flow (Bartosik, 2010).

The gravitational term, calculated by Equation (9), was subtracted from the measured total pressure drop over the vertical test sections for each set of solid concentrations. Finally, we can convert experimental data for vertical flow into data for horizontal flow. The equation for axially-symmetrical horizontal slurry flow is following:

$$\frac{\Delta p}{\Delta x} = \frac{4\tau_w}{D} \tag{10}$$

or

$$i_m \rho_L g = \frac{4\tau_w}{D} \tag{11}$$

where $\Delta p/\Delta x$ in equation (10) represents pressure drop after subtraction of the gravitational term from the total pressure drop in a vertical up-ward slurry flow.

Taking into account the Bagnold concept, we can assume that the wall shear stress is a sum of the shear stresses at the pipe wall caused by two factors, namely: the *particle – wall* stress and the *liquid – wall* stress (Bagnold, 1954). So, Equation (11) can be written as:

$$i_m \rho_L g = \frac{4\left(\tau_L + \tau_P\right)}{D} \tag{12}$$

and considering carrier liquid flow only, i.e. the wall shear stress caused by solid particles is zero ($\tau_P = 0$), we can write Equation (12) for carrier liquid, as follows:

$$\tau_L = i_L \rho_L g \frac{D}{4} \tag{13}$$

Shook and Bartosik (1994) and next Bartosik (1996) took into account Bagnold's concept and they developed an equation describing *particle – wall* shear stress, as follows:

$$\tau_P = B_B\, \rho_P\, d_{50}^2\, \lambda_B^{1.5} \left(\frac{dU}{dy}\right)^2 \tag{14}$$

Using Equations (12), (13) and (14), we can write:

$$i_m \rho_L g = \frac{4}{D}\left[\frac{D}{4} i_L \rho_L g + B_B\, \rho_P\, d_{50}^2\, \lambda_B^{1.5} \left(\frac{dU}{dy}\right)^2\right] \tag{15}$$

Taking into account the Newtonian hypothesis, as:

$$\frac{dU}{dy}=\frac{\tau_L}{\mu_L} \tag{16}$$

it is possible to obtain the following equation describing frictional head losses i_m in a vertical up-ward pipe flow for slurry with coarse particles:

$$i_m \rho_L g=\frac{4}{D}\left[\frac{D}{4} i_L \rho_L g+B_B\, \rho_P\, d_{50}^2\, \lambda_B^{1.5}\left(\frac{\tau_L}{\mu_L}\right)^2\right] \tag{17}$$

It must be emphasized that Equation (17) fails in predictions of frictional head loss i_m in a vertical up-ward slurry flow with coarse particles. It is due to the fact that constant B_B is strongly depending on a pipe diameter, which Equation (14) does not include. Bartosik (1996) found that if B_B is divided by the square of pipe diameter D^2, the function B_B/D^2 smoothly depends on Reynolds number. Such a modification, called *linearization*, requires changes in Equations (14) and (17), which final form, can be written as follows:

$$\tau_P=K_B\, \rho_P\, d_{50}^2\, \lambda_B^{1.5}\, D^2\left(\frac{dU}{dy}\right)^2 \tag{18}$$

and finally:

$$i_m=i_L\left(1+K_B\, \rho_P\, d_{50}^2\, \lambda_B^{1.5} i_L\, \rho_L\, g\, D^3\, \frac{1}{4\mu_L^2}\right) \tag{19}$$

where the function K_B is as follows:

$$K_B=\frac{B_B}{D^2} \tag{20}$$

The function K_B was designated on the basis of measured frictional head loss in the range of mean solid particles from 1.4 mm to 3.4 mm and for the volumetric solid concentrations from 0 to 0.45 and for two different pipe diameters. It was found that the K_B function depends on the Reynolds number, as follows:

$$K_B=8.3018\cdot 10^7\cdot Re_L^{-2.317} \tag{21}$$

while the Reynolds number is defined like for a carrier liquid flow, as follows:

$$Re_L=\frac{\rho_L U_b D}{\mu_L} \tag{22}$$

Linear concentration λ_B, which appears in Equations (14), (15) and (17) – (19), represents the ratio of the particle diameter to the mean distance between the particles and is expressed in terms of volumetric concentration C_V, and is as follows (Shook and Bartosik, 1994):

$$\lambda_B=\frac{1}{\left(\frac{C_{\max}}{C_V}\right)^{1/3}-1} \tag{23}$$

Finally, Equation (19), together with complementary relations (21) – (23), describes frictional head losses in a vertical up-ward slurry flow with coarse particles. The mathematical model is dedicated to predict frictional head loss in a smooth pipe in the range of mean particle diameters from 1.4 to 3.4 mm, and solid concentrations by volume from 0.1 to 0.45, and for narrowly sized solid particles with spherical, cylindrical and cubic shapes. It must be mentioned, however, that beyond the above parameters, the comparison of predictions with measurements were not made.

It is interesting to see how predicted friction factor for coarse slurries, converted from vertical to horizontal one, matches the measurements as we know that the friction factor is very sensitive to Reynolds number. Taking into account the general equation for the friction factor for slurry flow, described as:

$$\lambda_m=\frac{8\tau_w}{\rho_m U_b^2} \tag{24}$$

and using Equation (11) we can obtain a relation for the friction factor, as follows:

$$\lambda_m=\frac{2 i_m\, \rho_L\, g\, D}{\rho_m U_b^2} \tag{25}$$

Equation (25) together with Equation (19) will be used to predict friction factor λ_m for the vertical up-ward flow of slurry with coarse particles.

The validation of friction factor predictions

Predictions of friction factor λ_m for a turbulent vertical up-ward flow of slurry with coarse particles were performed for two pipe diameters and for three different solid particle diameters and for broad range of solid concentrations. The predicted friction factor was calculated using Equation (25), however, frictional head loss i_m in this equation was calculated using Equation (19). The predicted friction factor was compared with the measurements. The measurements were made in vertical up-ward recirculating flow loop described by Shook and Bartosik (1994) and Bartosik (2009). Parameters used for the validation of the friction factor predictions are collected in Table.1.

Table 1. Summary of the parameters used for the validation of the friction factor.

D m	d_{50} mm	ρ_P kg/m^3	C_V –
0.026	1.5	1045	0.10 – 0.40
	1.5	2650	0.10 – 0.30
	2.8	1045	0.20 – 0.45
	3.4	1400	0.20 – 0.40
0.040	2.8	1045	0.20 – 0.45
	3.4	1400	0.20 – 0.40

Each figure listed below, representing predicted and measured friction factor, possesses a referenced solid line, which represents the friction factor for carrier liquid (water). The friction factor for carrier liquid (water) was calculated using the Churchill formula (Churchill, 1977), assuming that inner pipe wall is smooth, and is expressed by Equations (26) – (28).

$$\lambda=8\left[\left(\frac{8}{Re_L}\right)^{12}+\left(A+B\right)^{-1.5}\right]^{\frac{1}{12}} \tag{26}$$

$$A = \left\{ 2.457 \ln \left[\left(\left(\frac{7}{Re_L} \right)^{0.9} + 0.27 \frac{\varepsilon}{D} \right)^{-1} \right] \right\}^{16} \tag{27}$$

$$B = \left(\frac{37530}{Re_L} \right)^{16} \tag{28}$$

Reynolds number in Equations (26) – (28) was calculated for water, in accordance with Equation (22), for temperature equal 25ºC, which is in accordance with performed experiments. The relative error, expressed by Equation (29), was calculated at the same Reynolds number for measurement and prediction.

$$Error = \frac{|(\lambda_m)_{\exp} - (\lambda_m)_{\mathrm{PRD}}|}{(\lambda_m)_{\mathrm{EXP}}} \cdot 100\% \tag{29}$$

The predicted versus the measured friction factor λ_m for polystyrene slurry in 0.026 m inner pipe diameter is presented in Fig. 1.

Fig. 2 shows that the experimental data for Cv = 0.30 are clearly below the friction factor for water. Analysing the data presented in Fig. 2, one can say that the highest relative error in predictions of friction factor λ_m is for C_V = 0.30 and Re_L = 185,000 and equals to 13.5%. This means that the model over-predict the friction factor. To analyse the data presented in Fig. 2, we have to consider Equation (25). Taking into account Equation (25) it is possible to set up condition when the frictional head loss for slurry should be lower than for water. Using Equation (25) for slurry and also for water, one can write:

$$\frac{i_m}{i_L} = \frac{\lambda_m}{\lambda_L} \frac{\rho_m}{\rho_L} \tag{30}$$

and finally, we can get the following condition:

$$i_m < i_L \iff \lambda_m S_m < \lambda_L \tag{31}$$

The ratio of slurry to liquid density, for data presented in Fig. 2, is equal S_m = 1.014. Let's consider the measured point of slurry friction factor for the highest Reynolds number and for C_V = 0.3, which is: λ_m = 0.01468 for Re = 185,000 – see Fig.2. For the same Reynolds number, the friction factor for water is λ_L = 0.0158. So, taking into account condition (31), we get: $\lambda_m S_m$ = 0.0149. It is seen that in this particular case the following condition exists $\lambda_m S_m < \lambda_L$. This means that frictional head loss for slurry should be lower than for water. To demonstrate that it is right, Fig. 3 shows experimental data of frictional head loss for the same slurry as presented in Fig. 2.

It is seen in Fig. 3 that the frictional head losses for slurry with C_V = 0.30 is lower than for water, which is in accordance with condition (31).

It is interesting to compare the predictions with the measurements for the same pipe and the same mean particle diameter, as presented in Fig. 2, but for sand slurry. In this case the particle density is about 2.6 times larger compared to polystyrene particles, presented in Fig. 2. Measurements and predictions presented in Fig. 4 explicitly proved that almost all points of friction factor are below data for water. In accordance with Equation (25) we can expect that if slurry density increases, the slurry friction factor should decrease. Nevertheless, it is surprising that friction factor λ_m is much below data for carrier liquid, which is pronounced for C_V = 0.20 and 0.30.

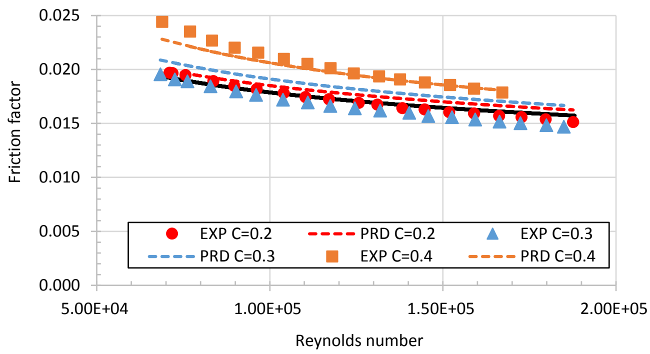

Fig. 2. Predicted versus measured friction factor λ_m in 0.026 m pipe for polystyrene slurry; d_{50} = 1.5 mm; ρ_P = 1045 kg/m^3.

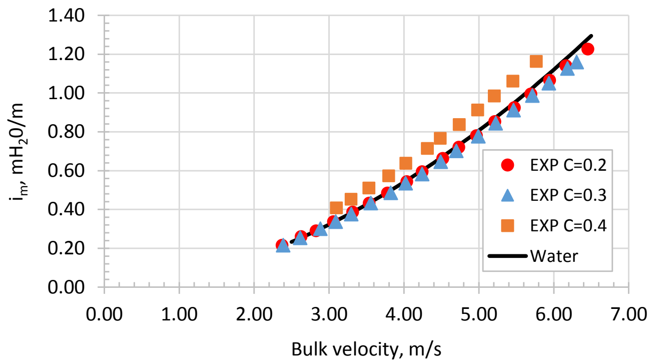

Fig. 3. Measured frictional head loss for slurry flow. Polystyrene particles: d_{50} = 1.5 mm, ρ_P = 1045 kg/m^3, pipe diameter D = 0.26 mm (Shook and Bartosik, 1994).

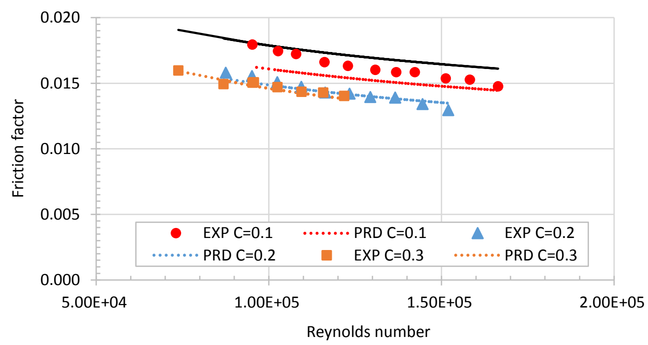

Fig. 4. Predicted versus measured friction factor λ_m in 0.026 m pipe for sand slurry; d_{50} = 1.5 mm; ρ_P = 2650 kg/m^3.

The results shown in Fig. 4 confirmed a fairly good agreement between the predicted and the measured friction factor λ_m if solid concentration was at least 0.20 by volume. The highest discrepancy appeared at C = 0.10 and Re_L = 95,200 and the relative error was about 9.4%. Considering condition, expressed by Equation (31), it can be found that in this case the frictional head loss should be slightly higher than for water.

With particle diameter increase we expected that the *particle – wall* interaction will increase too. The results shown in Fig. 5 confirmed high agreement between the predicted and the measured friction factor λ_m. Comparing the friction factor with the results presented in Fig. 2 and Fig. 5, it is obvious that increasing the mean solid particle diameter from 1.5 mm to 2.8 mm causes a substantial increase of friction factor λ_m. Results presented in Fig. 5 demonstrate that the highest relative error appears for C_V = 0.20 and Re_L = 180,000 and equals to 5%.

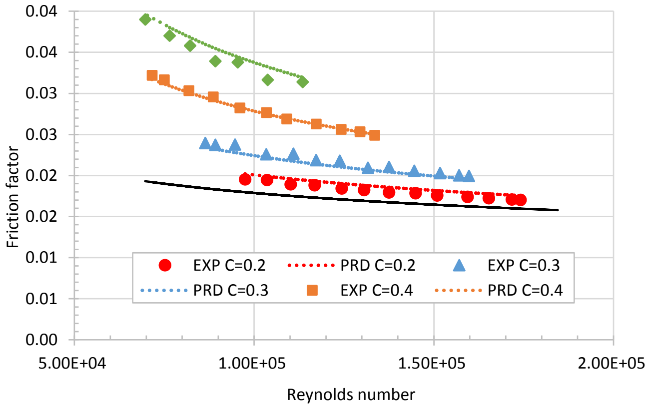

Fig. 5. Predicted versus measured friction factor in 0.026 m pipe for coarse polystyrene slurry; $d_{50} = 2.8$ mm; $\rho_P = 1045$ kg/m^3.

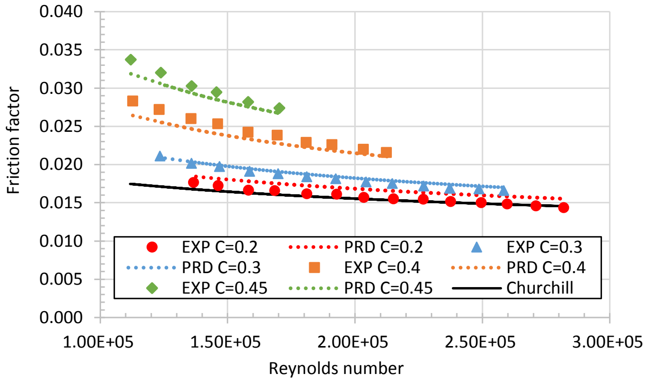

Fig. 7. Predicted versus measured friction factor in 0.04 m pipe for coarse polystyrene slurry; d_{50}=2.8 mm; ρ_P=1045 kg/m^3.

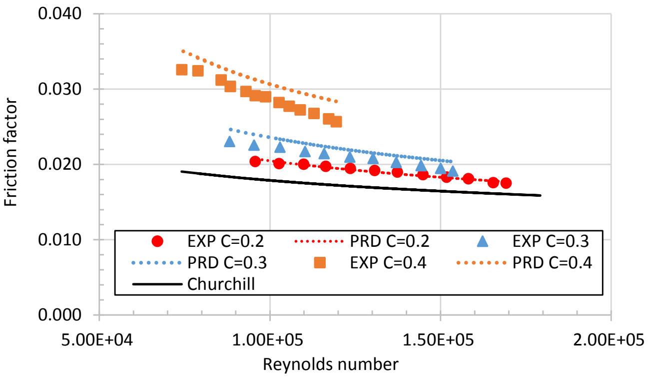

Fig. 6. Predicted versus measured friction factor in 0.026 m pipe for coarse PVC slurry; $d_{50} = 3.4$ mm; $\rho_P = 1400$ kg/m^3.

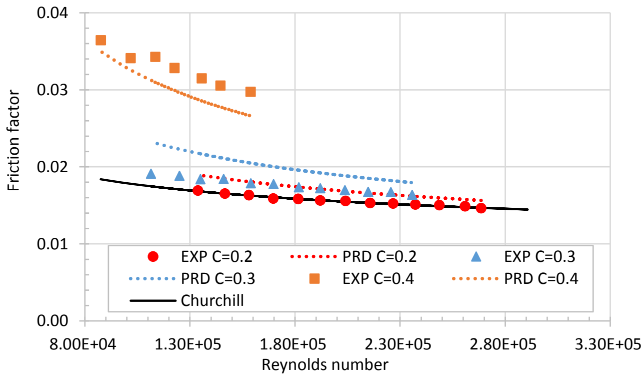

Fig. 8. Predicted versus measured friction factor in 0.04 m pipe for coarse polystyrene slurry; $d_{50} = 3.4$ mm; $\rho_P = 1400$ kg/m^3.

Predictions of λ_m for moderate solids density, like PVC, which is equal to 1400 kg/m^3, and for larger mean particle diameter, which is $d_{50} = 3.4$ mm, are presented in Fig. 6. In this case, it is seen that the model over predicts the friction factor mainly at $C_V = 0.40$, and the highest relative error is for Re_L = 119,000 and equals to 10.5%.

To ensure that the model is adequate for predictions of friction factor for coarse slurry flow, for different pipe diameters, comparisons of the predictions and measurements of the slurry friction factor were made for pipe with an inner diameter of 0.04 m. Results of friction factor for coarse polystyrene slurry with particles density 1045 kg/m^3 and mean particles diameter 2.8 mm are presented in Fig. 7. Predictions confirmed fairly good accuracy. The highest relative error exists for $C_V = 0.20$ and $Re_L = 282{,}000$ and equals to 8%.

The predicted and measured friction factor λ_m for coarse PVC slurry in pipe of $D = 0.04$ m was presented in Fig. 8. It is seen that the model over predicts the friction factor, especially for $C_V = 0.30$ and 0.40. In this particular case, the relative error is highest compared to all the presented figures thus far. The highest relative error is for $C_V = 0.40$ and $Re_L = 111{,}000$ and equals to 21.5%.

In conclusion, one can say that all the data presented in Fig. 2 and Fig. 4 – Fig. 8 showed that the predicted friction factor λ_m qualitatively follows the experimental data fairly well, although in some cases, like in Fig. 2 and Fig. 4, the friction factor was significantly below friction for water. Predicted friction factor confirmed that Equations (19) and (25) gave good qualitative and quantitative results for slurries presented in this study, except predictions for PVC, especially for pipe of inner diameter 0.04 m, where the highest discrepancy reaches 21.5%.

Comprehensive simulations of the dependence of particle diameter, solid concentration, solid particle density, and Reynolds number on *particle – wall* shear stresses were presented by Bartosik (2010).

DISCUSSION

A comparison of the predicted and measured friction factor λ_m for slurry flow with coarse particles demonstrate that the size of solid particles has a substantial influence on the friction. It is still not fully understood how slurry flow with coarse particles has a friction factor lower than for water flow, which was demonstrated in Fig. 2 and Fig. 4 for polystyrene and sand with $d_{50} = 1.5$ mm. Such results are in accordance with the measured frictional head loss for polystyrene and sand particles of d_{50} = 1.5 mm (Shook and Bartosik, 1994; Bartosik 2009). Shook and Bartosik (1994) subtracted the gravitational term from measured i_m values in vertical upward flow for polystyrene slurry with $d_{50} = 1.5$ mm and demonstrated that after the subtraction of the gravitational term, the frictional head loss for polystyrene slurry is lower than for water, which is presented in Fig.3. Such results suggest that damping of turbulence appears.

Bartosik (2009) subtracted the gravitational term from measured i_m values in vertical upward flow for sand slurry with $d_{50} = 1.5$ mm and demonstrated that after the subtraction of the gravitational term, the frictional head loss for sand slurry is slightly above that for water flow; even the slurry density was 50% higher than the water density (for $C_V = 0.3$). To illustrate this Fig. 9 presents measured frictional head loss for such slurry. Experiments were made for vertical up-ward flow, however, presented data were obtained in accordance with Equation (11), after subtraction of gravitational term, expressed by Equation (9), from the total pressure drop, expressed by Equation (7). Coloured dashed line in Fig. 9, represents calculation of slurry frictional head loss for $C_V = 0.2$ using Equation (32). We expect that for solid concentration equal, for instance 0.2, which gives $S_m = 1.33$, the i_m should be much higher than it is. For instance, when bulk velocity equals to 4 m/s the i_m should be 23% higher than experiments proved. It is very probable again that in such a case damping of turbulence appears.

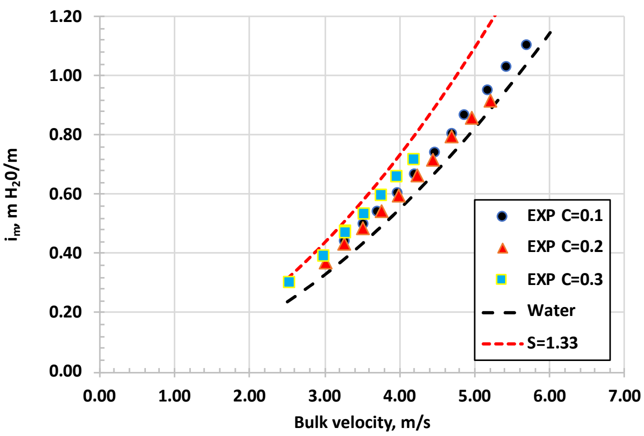

Fig. 9. Measured frictional head loss for slurry flow. Sand particles: $d_{50} = 1.5$ mm, $\rho_P = 2650$ kg/m^3, pipe diameter $D = 0.26$ mm (Bartosik, 2009).

$$i_m = S_m \, i_L \tag{32}$$

The measured and predicted friction factor for slurry flow with particle diameters equal to 2.8 and 3.4 mm demonstrated that the friction factor is significantly above friction for water, especially in the range of solid concentrations $0.2 < C_v \leq 0.3$ – see Fig. 5–Fig. 8. However, for pipe diameter $D = 0.04$ m and low solids concentration ($C_V = 0.20$) the friction factor was very close to friction for water – Fig. 7 and Fig. 8. In all the above cases, the friction factor decreases as Reynolds number increases.

Looking for an explanation of low friction factor, which is pronounced for mean particle diameter of polystyrene and sand, equal to 1.5 mm, we have to consider the region close to a pipe wall, as this region determines friction in a flow. If turbulent flow is considered, we know that the viscous sublayer and buffer layer play a crucial role in production of the friction. In those two regions the velocity gradient is highest and in buffer layer the intensity of turbulence achieves maximum values. It seems to be reasonable to assume that *lift forces* and damping of turbulence are additional important factors affecting shear stresses. Particle diameter, solid concentration, and particle density are important components influencing *lift forces* and turbulence damping, as was stated in Introduction.

Summer et al. (1990) are among others who have shown that the concentration distribution of the solids in a slurry flow in vertical pipe depends upon particle size and solids concentration. The results of their experiments showed that the tendency of particle concentrations to decrease near a pipe wall was independent of bulk velocity and pipe diameter but increased with bulk concentration. At high concentrations and larger particle diameters the particles tend to move towards the centre of the pipe with the result being a decreased concentration region near the wall of the pipe.

Several researchers emphasized the importance of *lift forces* acting on solid particles traveling near a pipe wall (Kaushal and Tomita, 2007; Matousek, 2009; Vlasak et al., 2013; Wilson, 2003). Some of researchers emphasized that the off wall forces are much more effective for medium than for coarse particles (Matousek, 2009; Sumner, 1992). If we assume that the existence of *lift forces* pushes particles away from a pipe wall, we could expect that the lowest possible frictional head loss would be that for the carrier liquid. However, we demonstrated it is not true, because the slurry frictional head loss is below water in some cases. Therefore, hypothetically one can say that despite the *lift forces*, there are other additional factors, which are responsible for damping of turbulence at a pipe wall. Of course, the damping of turbulence depends on several factors, among them there is size of solid particles, particles density, solid concentration, Reynolds number etc. Hypothetically, one can say that the damping of turbulence could be an important drive force, which causes that viscous sublayer becomes thicker. A similar observation was made by Wilson and Thomas (1985), however, for fine dispersive slurry flows, which exhibit non-Newtonian behaviour. To validate such hypothesis access to measurements of fluctuating components of velocity of carrier liquid and solid particles at a pipe wall is required.

CONCLUSIONS

The validation of predictions of friction factor in vertical pipes for slurry flow with coarse particles was made using the mathematical model, which contains Equations (19) and (25). Analysing Equation (19) it is seen that the frictional head loss strongly depends on solid particle diameter and next on solid concentration and solid density. The mathematical model assumes that two major factors affect the total wall shear stress. The first one is due to the *particle – wall* shear stress and the second is due to the wall shear stress caused by carrier liquid. As was mentioned, in such a case *lift forces* and damping of turbulence could exist. As the model predicts the friction factor fairly well, it is possible that the effect of *lift forces* and turbulence damping is included in the K_B function, which depends on Reynolds number. The K_B function was tuned on the basis of experimental data. Therefore, the function describes other phenomena, which the model in its nature does not assumes.

The mathematical model emphasised importance of mean particle diameter, solid concentration, particle density, carrier liquid viscosity and bulk velocity. It was found that mathematical model overpredicts friction factor for slurry flow with mean particles diameter 3.4 mm in pipe of inner diameter $D = 0.04$ m – see Fig. 8. In this particular case the highest relative error was 21.5%. Predictions of slurry friction factor for polystyrene and sand particles demonstrate fairly good agreement with measurements.

Results of predictions indicate that the assumption made in the mathematical model that the total shear stresses depend on two factors only, i.e. *particle – wall* shear stress and *liquid – wall* shear stress, is not sufficient.

The mathematical model of friction factor λ_m could be improved if more complex functions than (12) and (21) will be considered. Such functions could include turbulence damping, which is one of the most important factors of the transport phenomena.

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NOMENCLATURE

A	– cross section of a pipe, m^2 or variable in Churchill formula (–)
B	– variable in Churchill formula, –
B_B	– dimensionless function in Bagnold concept, –
C_V	– solids concentration by volume, –
$C_{\max}$	– maximum possible solids concentration in a pipe, –
d_{50}	– mean particle diameter, mm
D	– inner pipe diameter, m
EXP	– experimental data
g	– acceleration due to gravity, m/s^2
h	– elevation above datum, m
i_L	– frictional head loss for liquid, m/m
i_m	– frictional head loss for slurry, m/m
K_B	– empirical function, m^{-2}
p	– static pressure, Pa
PRD	– predicted data

r – distance from symmetry axis, m
Re_L – Reynolds number for carrier liquid phase, –
S_m – ratio of slurry to liquid density, –
S_ρ – ratio of solid to liquid density, –
U – axial component of velocity (bulk velocity), m/s
U_b – bulk velocity of flowing medium (slurry or water), m/s
V – radial component of velocity, m/s
W – circumferential component of velocity, m/s
x – main flow direction, m
y – radial coordinate, m
Δ – difference
λ_B – dimensionless linear concentration, –
λ_m – friction factor for slurry, –
μ_L – liquid viscosity, Pa s
ρ_L – liquid density, kg/m^3
ρ_m – slurry density, kg/m^3
ρ_P – particle density, kg/m^3
τ_L – liquid component of wall shear stress, Pa
τ_P – particle component of wall shear stress, Pa
τ_w – wall shear stress, Pa

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